Firm Expansion, Size Spillovers and Market Dominance in Retail Chain Dynamics*

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Abstract

We develop and estimate a dynamic game of strategic firm expansion and contraction decisions to study the role of firm size on future profitability and market dominance. Modeling firm size is important because retail chain dynamics are more richly driven by expansion and contraction than de novo entry or permanent exit. Additionally, anticipated size spillovers may influence the strategies of forward looking firms making it difficult to analyze the effects of size without explicitly accounting for these in the expectations and, hence, decisions of firms. Expansion may also be profitable for some firms while detrimental for others. Thus, we explicitly model and allow for heterogeneity in the dynamic link between firm size and profits as well as potential for persistent brand effects through a firm-specific unobservable. As a methodological contribution, we surmount the hurdle of estimating the model by extending the Bajari, Benkard and Levin (2007) two-step procedure that circumvents solving the game. The first stage combines semi-parametric conditional choice probability estimation with a particle filter to integrate out the serially correlated unobservables. The second stage uses a forward simulation approach to estimate the payoff parameters. Data on Canadian hamburger chains from their inception in 1970 to 2005 provides evidence of firm-specific heterogeneity in brand effects, size spillovers and persistence in profitability. This heterogeneous dynamic linkage shows how McDonald's becomes dominant and other chains falter as they evolve, thus affecting market structure and industry concentration.

Keywords: Dynamic discrete choice, firm expansion, size spillovers, market dominance, retail chains, persistence in profits, particle filter, serial correlation.

1 Introduction

The strategic decision of a firm to expand or contract is inherently dynamic with long term implications for not only its market outcomes but also those of its rivals and, hence, industry structure and evolution. We develop and estimate a dynamic game of strategic firm expansion and contraction decisions to study the role of firm size on future profitability and, thus, market dominance and structure. The anticipated effects of firm size on future profitability, i.e., size spillovers, may influence the strategies of forward looking firms. This may in turn affect the evolution of market structure. For example, firms may "over-expand" in periods when they expect positive spillovers (e.g., Shen and Villas-Boas 2010). Moreover, these spillovers may not even be realized but would still affect industry evolution. Therefore, it may not be possible to analyze the effects of size without explicitly accounting for these in the expectations and, hence, decisions of firms. Expansion may also be profitable for some firms while detrimental for others. Consequently, we explicitly model the potential for size spillovers and firm specific heterogeneity within these. However, this presents a methodological obstacle, especially when some components of profits are unobserved to the researcher and serially persistent over time due to heterogeneity in size spillovers and firm specific effects. We also provide an estimation procedure to address this challenge.

In particular, our work is motivated by three issues. First, it is a stylized fact that the dynamics of retail market structure are more richly driven by expansion and contraction than de novo entry or permanent exit (Hanner et al., 2011). Most empirical models ignore these rich underlying dynamics and focus on entry and exit. Second, there may be a heterogeneous relationship between firm size and profitability, i.e., some firms may grow and become more profitable (e.g., Chandler, 1990, Huff and Robinson 1994, Robinson and Min 2002) whereas others may find it detrimental to expand (e.g., Fisher, McGowan, and Greenwood, 1983, Golder and Tellis, 1993, Kalyanaram, Robinson, and Urban 1995, Min, Kalwani and Robinson 2006, Sutton 2007).[1] Furthermore, even within a given firm this relationship may vary with firm size. Demonstrating and understanding this relationship is an empirical matter. It is important not just because of practical strategic considerations for firms when deciding on expansion or contraction but also for public policy, e.g.,

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[1]There is ample evidence of big firms such as Walmart, Amazon or Starbucks getting even bigger, see e.g., (*The Economist*, 2012). In fact, sometimes expansion in itself is a key performance index for firms, e.g., recently a key priority for the Four Seasons hotel chain has been to adopt a location growth strategy (*The Economist*, 2013). On the other hand, there are many examples of firms that were once dominant but have since experienced various stages of decline, e.g., Kmart, Circuit City, and Blockbuster. More interestingly, there is also evidence of firms that continue to be dominant but have found that rapid firm expansion may come at the cost of quality, and thus, profitability. Indeed, Toyota's focus on size and growth has been blamed as one of the main reasons behind recent overlooked safety issues with its cars (see e.g., *BBC News*, 2010, Cole, 2011).

whether some firms can become dominant relegating others to the fringe, and the consequences of this for market structure. Moreover, if a dynamic link exists between firm size and profitability then it implies that a forward-looking strategic firm will incorporate such size spillovers in its decision to expand or contract, which has implications for estimating the effect of a firm's size on its payoffs and decisions. Third, static and dynamic games conventionally account for unobserved (to the researcher) firm specific heterogeneity in profits using a time invariant fixed effect, if at all. In a dynamic setting firm specific unobserved heterogeneity in profits may display serial persistence and also evolve based on the history of the firm's actions. Accounting for such time varying unobserved heterogeneity in profits when estimating a dynamic game presents a severe econometric challenge.

To our knowledge these three issues have not been studied in a unified setting in the existing literature on dynamic oligopoly models. Thus, we develop and estimate a dynamic game based on micro foundations of strategic interaction between forward looking firms that recognize that firm size may affect future profitability and competitiveness. In the model, firms choose to expand or contract and firm size is endogenous. Size, in turn, has spillovers on a firm's future profitability, and thereby on its relative dominance[2] and market structure. Additionally, the model allows for heterogeneity in the dynamic link between firm size and future profitability either due to inter-temporal spillovers of size or persistence in past profitability shocks. This link is incorporated through a firm specific unobservable (to the researcher) variable.

Given the current state of econometric methods (see e.g., Ackerberg, Benkard, Berry, and Pakes, 2007) it remains very difficult to estimate such a dynamic game that contains a firm specific variable that is potentially unobserved (to the researcher), serially correlated and subject to endogenous feedback. The hurdle of estimating the model is surmounted by extending a two-step procedure that doesn't require solving the game, that was proposed by Bajari, Benkard and Levin (2007), building in turn on the work of Hotz and Miller (1993) and Hotz, et al. (1994). The first stage combines semi-parametric conditional choice probability estimation with a particle filter, or sequential Monte Carlo procedure, to integrate out the serially correlated unobservables. The second stage uses a forward simulation approach to estimate payoff parameters.

As a step to understanding the dynamic linkages between firm size, profitability and market dominance, our work builds on various literatures. The cornerstone of our work is the literature on estimating static and dynamic games of entry (e.g., Bresnahan and Reiss, 1991a,1991b, Berry,

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[2]Given the lack of a single agreed upon definition for what constitutes a fringe or dominant firm, in what follows we define a firm to be dominant if it has larger than equal market share.

1992, Ericson and Pakes, 1995, Scott Morton, 1999, Seim, 2006, Pakes, Ostrovsky, and Berry 2007, Aguirregabiria and Mira, 2007, Pesendorfer and Schmidt-Dengler, 2008, Ellickson and Misra, 2008, Zhu and Singh, 2009, Zhu, Singh and Manuszak 2009, Vitorino, 2012, Datta and Sudhir, 2013, Orhun, 2013, Collard-Wexler, 2013, Shen, 2014, Igami and Yang, 2015) that has used entry and exit decisions of firms to estimate the primitive parameters of the payoffs, and hence, infer industry conduct. In particular, we extend the workhorse model of entry (e.g., Bresnahan and Reiss, 1991b, Berry, 1992) to a dynamic setting to allow for firm expansion or contraction and endogenous evolution of firm size.[3]

To model the dynamic link between firm size and future profitability using inter-temporal size spillovers and persistence in past profitability, we borrow from the literature on firm size, capacity expansion and industry dynamics (e.g., Lucas, 1978; Rao and Rutenberg, 1979; Jovanovic, 1982; Hopenhayn, 1992; Boulding and Staelin, 1990; Shen and Villas-Boas, 2010). In particular, we model a firm level profitability shock that follows a Markov process (e.g., Ijiri and Simon, 1967, Jovanovic, 1982, Hopenhayn, 1992). Further, we endogenize this shock to past firm decisions as in Ericson and Pakes (1995). They treat the shocks as endogenous to firm R&D decisions while in our case we treat the profitability shock as a function of firm size. This is predicated on the fact that firm size expansion via store proliferation is a key investment and strategic decision for retail chains, and a proxy for their experience and familiarity with the market.[4] Although, given our data we cannot disentangle the underlying sources of the firm size spillovers,[5] such as learning by doing, or economies of scale or scope, our empirical implementation of the link between profitability and size spillovers is related to the long literature on learning by doing, e.g., Arrow (1962), Bass (1980), Dolan and Jeuland (1981), Rao and Bass (1985) Benkard (2000, 2004), Besanko et al. (2010), Bollinger and Gillingham (2013). Moreover, as in Benkard (2000) and Besanko et al. (2010) we allow for persistence of the past profitability shocks, which may loosely be interpreted as institutional memory.[6] Our empirical specification of the controlled stochastic process that defines

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[3]Collard-Wexler (2013) also allows for firm size to be endogenous but in contrast to his model where firms may choose to be "small," "medium," or "large," our model allows for a much finer choice of firm size.

[4]A common practice now for retail chains is to purchase the land that houses their stores (Love, 1995). In that sense, expansion or contraction can be seen as strategic real estate investment decisions.

[5]One may also broadly relate our research in to the dynamic link between firm size and unobserved profitability to the extensive literature on estimating Total Factor Productivity (TFP), where this measure can be thought of as the residual (and unobserved) component that explains variation in output after relevant inputs have been taken into account. TFP (or the Solow residual) is often considered to be a measure of long run technological change or technological productivity of an economy. See Syverson (2011) for a recent, comprehensive survey of this literature.

[6]Previous research has generated mixed findings about institutional memory or organizational forgetting. For instance, Argote, Beckman, and Epple (1990), Darr, Argote, and Epple (1995), Epple, Argote and Devadas (1991), and Epple, Argote and Murphy (1996) provide evidence in favor of depreciation with data from shipbuilding, pizza

the firm level spillover can also be broadly linked to the literature on profitability dynamics (e.g., Hall and Weiss, 1967; Schmalensee, 1989, Waring 1996). However, in contrast, our controlled stochastic spillover process generates an *endogenous* vector of serially correlated, unobserved firm-specific state variables in the dynamic game.

Our work is related to the literature on using particle filters, or sequential Monte Carlo, to control for serially correlated unobserved heterogeneity in dynamic models. A special case of this is the GHK simulator (see e.g., Keane, 1994, Erdem and Keane, 1995) that arises for a particular choice of the Gaussian distribution. Monte Carlo based Bayesian methods have also been used in the estimation of single agent dynamic discrete choice models, e.g., Imai, Jain and Ching (2009) and Norets (2009). It should be noted however that we use particle filters, or sequential Monte Carlo methods, in a frequentist framework (e.g., Chernozhukov and Hong, 2003). More recently, particle filters have also been used to estimate dynamic equilibrium models (e.g., Fernandez-Villaverde and Rubio-Ramirez, 2007, Blevins, 2015, Gallant, Hong and Khwaja, 2014, 2015). We depart from this literature in some important ways. We allow for explicit strategic interaction among agents in the form of a dynamic game. Our two-step estimation method extends Blevins (2015) and allows for endogenous feedback from past actions in the serially correlated firm specific unobservable state. Moreover, our dynamic oligopoly model of firm size dynamics is estimated in a way that is computationally easier to implement than the "full solution," or nested fixed point, approach adopted by Gallant, Hong and Khwaja (2014, 2015).

Our paper also provides a bridge between dynamic oligopoly models that explicitly incorporate strategic interactions between forward-looking firms and the literature modeling serially correlated unobservable variables using either Kalman filters in dynamic linear settings or particle filters in reduced form models of firm decisions (e.g., Naik, Raman, and Winer 2005, Sriram, Chintagunta, and Neelamegham 2006, Sriram and Kalwani 2007, Jap and Naik 2008, Bass et al. 2007, Bruce 2008). For example, Pancras, Sriram, and Kumar (2012) develop a demand model with latent goodwill dynamics, location endogeneity, and spatial competition between retail outlets. The latent goodwill state follows an AR(1) process and the model is estimated using a Kalman filter. In contrast, our model is nonlinear in the latent profitability state due to the forward-looking optimal decision making behavior of agents, which gives rise to a dynamic programming problem, and

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chains, truck production, and automotive assembly respectively. In contrast, Thompson (2007) finds a weaker depreciation effect in the shipbuilding industry once sufficient controls are included in the analysis. Sorenson (2003) provides evidence from the computer workstation manufacturing industry on heterogeneity in organizational learning depending on internal firm and external market structure.

endogenous feedback from the optimal decision to future values of the latent profitability state. This is the methodological challenge we face and our contribution is to develop a two-step approach to estimate nonlinear, dynamic discrete (multinomial or ordered) choice models with serially-correlated latent state variables following an arbitrary parametric law of motion.

Our method is also related to the work of Arcidiacono and Miller (2011), who provide an empirical framework via expectation-maximization for integrating out persistent unobserved heterogeneity in dynamic discrete choice models. However, there are some key differences: (i) our method allows for the serially correlated unobservable to have continuous support, (ii) be firm (or agent) specific as opposed to market specific, and (iii) endogenous to past agent actions.[7]

While our work focuses on within-firm size spillovers, there has been other related work that looks at industry-wide or inter-firm spillovers, and "learning from others (consumers or firms)" or "aggregate learning," e.g., Ching (2010), Shen (2014), Shen and Xiao (2014), Toivanen and Waterson (2005), and Yang (2015a). While such factors are likely relevant in many retail industries, it is unlikely they play a dominant role in explaining firm dynamics in our setting. In fact, using a subset of the same raw data and focusing on the city of Toronto, Yang (2015a) finds that learning from others accounts for at most 5% of the retail clustering observed in small neighborhood markets.

Using data on Canadian hamburger retail chains from their inception in 1970 to 2005, we study the decision to expand or contract for A&W, Burger King, Harvey's, McDonald's, and Wendy's across all Canadian cities. This setting provides us a suitable laboratory for studying the relationship between firm size and market dominance.Firstly, the time period we study captures comprehensive dynamics on the extensive margin, as is clearly reflected in the raw data patterns.[8] Secondly, each outlet that is constructed by a retail chain is nearly identical, in terms of outlet size and product offerings. Therefore, the volatility in expansion and contraction helps map out the spillovers of each firm's size over time. The estimated model generates a number of insights. First, we demonstrate that our baseline model, one that incorporates a serially correlated unobserved profitability component, fits the data better than alternative models that ignore such effects. Second, our estimates provide evidence of heterogeneity in brand effects, inter-temporal size spillovers

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[7]Conceivably, Arcidiacono and Miller's (2011) method could be extended to allow for agent specific unobservables but this might lead to a proliferation of parameters, e.g., if an agent specific transition matrix for the unobservables is required. Similarly, the transition matrix could potentially be allowed to depend on lagged choices of agents but we are unaware of such an implementation. Conversely, incorporating market specific unobservables instead of firm (or agent) specific unobservables is a straightforward special case of our set up.

[8]The active periods of expansion in our data are consistent with views expressed through personal communication with high-level real estate managers, from Harvey's, McDonald's, and Wendy's, who mentioned an overarching goal of outlet growth during the time interval we study.

and persistence in profitability across firms. These in turn affect the evolution of profits and, hence, expansion of different hamburger chains. Third, in particular, these affect McDonald's growth and dominance over time. Fourth, we show that the market dominance McDonald's enjoys is robust to various market and competitor level shocks. Overall, we find that this heterogeneous dynamic linkage shows how some firms become dominant and others falter as they expand, thus affecting market structure and industry concentration.

The setting of hamburger chains in Canada has also been the subject of other research. In particular, Igami and Yang (2015) also study expansion patterns in this industry.[9] Relative to their work, our model includes richer firm and market heterogeneity—both observed and unobserved— and focuses on industry dynamics in a broader sense across the entire country. Our analysis controls for more observed heterogeneity in three ways: we use data from all 31 cities in Canada, we explicitly consider the identity of each firm, and we include more time-varying explanatory variables. Igami and Yang (2015) focused on only the seven largest cities in Canada, they considered two observed firm types (McDonald's and other firms), and they only used data on population and income. Additionally, we use data on property values, minimum wage, Grey Cup hosting, and smoking regulations. Controlling more thoroughly for observed heterogeneity is important because the key substantive and methodological insights of our paper revolve around time-varying unobserved heterogeneity. Igami and Yang (2015) allow for three discrete, permanent types in their main specification using the approach of Arcidiacono and Miller (2011), which is based on the expectation-maximization (EM) algorithm. Our approach is based on novel particle filtering methods and allows for richer forms of unobserved heterogeneity in the form of continuous variables that vary over time and evolve both stochastically and endogenously. Finally, the focus of our paper is quite different from that of Igami and Yang (2015). They focus on cannibalization and preemption in a subset of very large cities and define markets to be small neighborhoods (each with a 0.5 mile radius) within those cities, in which these aspects are more pronounced. Our analysis focuses more broadly on firm expansion in city-sized markets and uses data from all Canadian cities. In these larger geographic areas we abstract away from the effects of cannibalization and preemption, which will necessarily be muted since new outlets may still be located at some distance from existing ones.[10]

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[9]We also note a few other papers that study the Canadian fast food industry. First, Yang (2015a) examines "aggregate learning" or learning from the decisions of other firms. In another paper, Yang (2015b) tests for preemption in firm strategies using a reduced form approach. Similar to Igami and Yang (2015), the focus of these papers is on much smaller geographic markets nested within metropolitan areas.

[10]An argument in favor of a broader market definition is that demand for fast food in one city is unlikely driven by

2 Data and Empirical Patterns

2.1 Market Characteristics

For our analysis, we study the retail chain outlet expansion and contraction patterns across all Canadian Census Metropolitan Areas (i.e., cities) from 1970 to 2005. The hamburger retail chains we focus on are the main players in the industry during that time period: A&W, Burger King, Harvey's, McDonald's, and Wendy's. In total, our panel covers 31 cities over 36 years for each of the 5 retail chains. We interpret each city as an isolated market. Cities are separated by distances of at least 60 km.[11] We choose this definition of market for two main reasons. First, one could make an argument, similar to Toivanen and Waterson (2005), that demand in one city's fast food is unlikely driven by residents in another city at least 60 km away. Secondly, while individual managers within each chain's real estate division have well-defined geographic jurisdictions, each city's headquarters has a real estate manager that is in charge of the chain's overall growth strategy for that city. With this definition of the market, in each year we observe how many new stores were added, how many existing stores there are, and how many existing stores were closed.

Table 1 provides a snapshot of the main variables used for estimation. Most retailers have about three outlets on average, across markets and over time, while McDonald's has about 12. Figure 1 displays the growth of the fast food industry as measured by the annual total number of outlets.[12]

The retail store entry and exit data in raw form was originally collected by Yang (2015a), who used historical archives of phone directories to track each and every outlet that was ever in operation in Canada. We augment this data with information from a number of sources. We add market characteristics obtained from the Canadian Census. In particular, we have characteristics that affect revenue, such as population and income, and characteristics that may affect the fixed costs, such as property value (as many retail chains purchase the land on which their restaurants reside). We also include region-specific minimum wage levels over time from the Human Resources

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residents in another city at least 60 km away (which is the minimum distance between any two cities in our data). On the other hand with a smaller market definition such as city blocks or neighborhoods, demand across these geographic markets may be correlated. For these reasons, a broad market definition has also been used in other past studies about retail chains, such as Shen and Xiao (2014) and Toivanen and Waterson (2005).

[11]In terms of driving time between two cities in our sample that are in closest proximity to one another (Toronto and Oshawa), one would need to drive at least 40 minutes.

[12]The literature typically refers to the share of revenues or sales as "market share." In the absence of data on sales or revenues, in what follows by "market share" of a brand we mean the proportion of stores that the firm i owns in market m at time t. We acknowledge that our use of the term market share is somewhat unconventional relative to the literature, but on the other hand it is not illogical within our context. So we use it rather than invent new terminology or use a more unwieldy phrase such as "the share of stores." Moreover, if we had data on revenues or sales, then our approach would yield exactly the same definition of market share as commonly understood in the literature.

Table 1: Summary Statistics

Variable	Mean	S.D.	Min.	Max.	Obs.
Annual number of A&W outlets per city	4.5	6.9	0	50	1116
Annual number of Burger King outlets per city	2.6	3.7	0	23	1116
Annual number of Harvey's outlets per city	2.9	5.9	0	54	1116
Annual number of McDonald's outlets per city	12.3	21.7	0	164	1116
Annual number of Wendy's outlets per city	2.3	3.5	0	23	1116
Annual change in number of A&W outlets	0.2	0.9	-7	13	1116
Annual change in number of Burger King outlets	0.2	0.7	-2	10	1116
Annual change in number of Harvey's outlets	0.2	1.0	-14	18	1116
Annual change in number of McDonald's outlets	0.8	2.1	-7	29	1116
Annual change in number of Wendy's outlets	0.2	0.6	-4	6	1116
Annual number of cities entered by A&W	1.0	1.9	0	8	1116
Annual number of cities entered by Burger King	0.9	1.6	0	7	1116
Annual number of cities entered by Harvey's	0.9	1.2	0	4	1116
Annual number of cities entered by McDonald's	0.9	1.9	0	8	1116
Annual number of cities entered by Wendy's	0.9	1.5	0	6	1116
Annual number of cities exited by A&W	0.2	0.5	0	2	1116
Annual number of cities exited by Burger King	0.1	0.3	0	1	1116
Annual number of cities exited by Harvey's	0.1	0.4	0	2	1116
Annual number of cities exited by McDonald's	0.0	0.0	0	0	1116
Annual number of cities exited by Wendy's	0.1	0.2	0	1	1116
HHI (based on number of outlets)	0.4	0.2	0.2	1.0	1076
Population (millions)	0.4	0.5	0.1	2.9	1116
Income (millions)	0.1	0	0	0.1	1116
Property value (millions)	0.3	0.1	0.1	0.8	1116
Minimum wage	4.6	1.9	1.1	8	1116
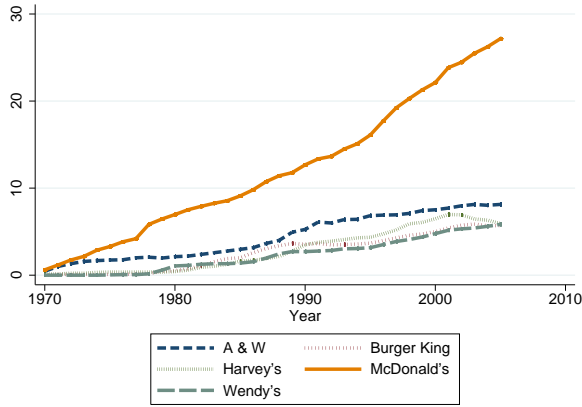
(a) Average number of outlets
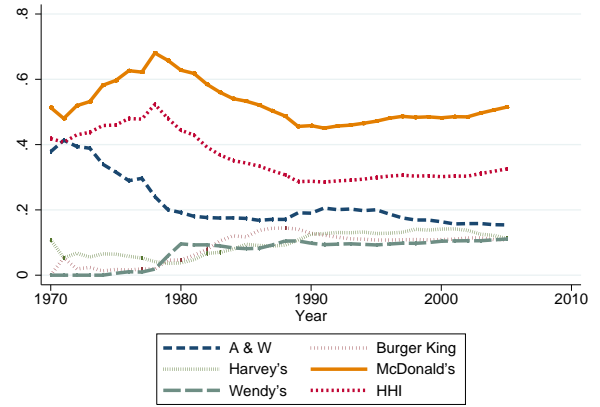
(b) Market share

Figure 1: Evolution of Market Structure Over Time

and Skills Development Canada online database. This provides an additional variable that controls for cost, as fast food chains often hire workers at or near minimum wage. We also include information about whether a city hosted the Canadian Football League's (CFL) Grey Cup championship tournament.[13] Finally, we use additional information about the roll-out of anti-smoking regulations across Canadian municipalities. From Shields (2007), we obtained the years in which each municipality introduced smoking by-laws that prohibit people from smoking in public places. In Canada, these regulations were first introduced in certain cities before being enacted more generally at the provincial level. This additional data is included in our analysis as past work has shown that smoking by-laws have an impact on the amount of food consumed in restaurants (Lewis, 2012).

2.2 Expansion and Contraction Patterns

To motivate our decision to focus on expansion and contraction in the dynamic oligopoly model, we present total counts of entry, exit, re-entry, and re-exit. Table 2 illustrates that while there is variation in these events across retail chains, there is disproportionately more expansion and contraction than pure entry and exit. This pattern is consistent with recent evidence that changes in market shares and industry structure among retailers are largely due to expansion and contraction by incumbents rather than de novo entry or permanent exit (Hanner et al., 2011).

To focus on a concrete example, consider A&W's experience in Abbotsford, British Columbia. It first entered the city in 1972, exited in 1975, re-entered in 1976, and exited again in 1984. A&W re-entered in 1988, and then expanded by one store per year in 1991 and 1992, followed by a contraction of two stores in 1993, expansion by one store per year in 1994 and 1995, contraction of two stores in 1996, expansion of two stores in 1997, contraction of one store in 1999, and expansion of one store in 2002. Hence, most of A&W's decisions after 1988 would not be captured by a model of entry and exit alone. Furthermore, modeling expansion and contraction patterns will identify dynamics otherwise left out by focusing solely on entry and exit, and the effects of these dynamics on market structure and its evolution.

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[13]This event is the Canadian equivalent to the National Football League's (NFL) Super Bowl. Each year, a city is selected to host the Grey Cup by a board of governors at the CFL. While this process is done through a bidding process to ensure that a certain level of revenue can be generated, the board tries to rotate the event across all member cities. The Grey Cup event draws in fans from all provinces, and is said to generate significant revenues for the host city (Johnstone, 2012). There may also be some long-run benefits in the form of improved infrastructure and construction of new facilities, as these investments are often conditions of the submitted bids. Refer to `http://cfldb.ca/faq/league/` for more details.

Table 2: Average Annual Expansion and Contraction by Chain.

Chain	Expansion	Contraction	Entry	Exit	Re-Entry	Re-Exit
A&W	237	78	25	4	4	1
Burger King	178	45	31	3	3	0
Harvey's	186	44	27	4	2	0
McDonald's	380	17	23	1	1	1
Wendy's	184	27	31	4	4	2

Table 3: Market Expansion Regressions

	(1) A&W	(2) Burger King	(3) Harvey's	(4) McDonald's	(5) Wendy's
Lagged A&W expansion	0.0475 (0.0809)				
Lagged Burger King expansion		0.0228 (0.0520)			
Lagged Harvey's expansion			0.0867*** (0.0252)		
Lagged McDonald's expansion				0.0245* (0.0116)	
Lagged Wendy's expansion					0.132* (0.0542)
Observations	1116	1116	1116	1116	1116
Controls	Yes	Yes	Yes	Yes	Yes
City dummies	Yes	Yes	Yes	Yes	Yes

Clustered standard errors (by market) in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

2.3 Persistence in Expansion

Our aim is to understand the dynamic link between firm size, profitability and market dominance and the persistence in profitability that this might imply. This persistence will also materialize through correlation between past and current firm expansion. For example, if increases in firm size lead to greater profitability and hence even more expansion, then there should be a positive correlation between past and current expansion. On the other hand if firm size is detrimental to profitability then there should be a negative correlation between past and current expansion. To study the relationship between firm size and subsequent market dominance, we first examine the raw data and then consider the linear regression of current expansion on lagged expansion.

First, Figure 2 provides evidence that suggests there is a positive relationship between current and past expansion. This link appears to be strongest for McDonald's. Second, the regression results reported in Table 3 are consistent with the graphical patterns. That is, we find evidence of a
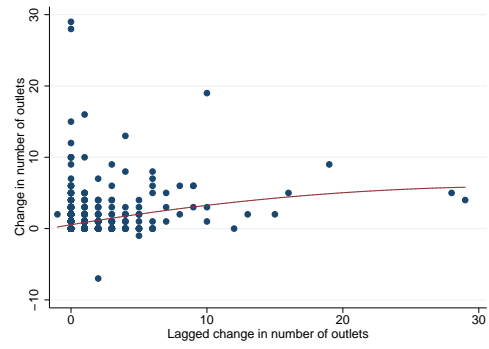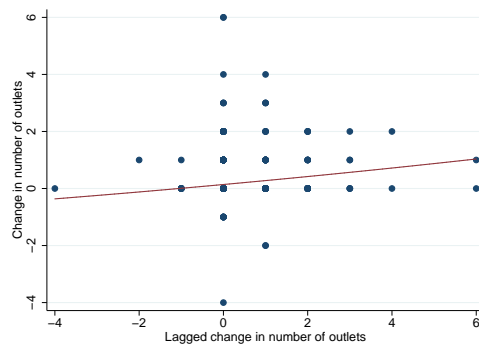
(a) A&W
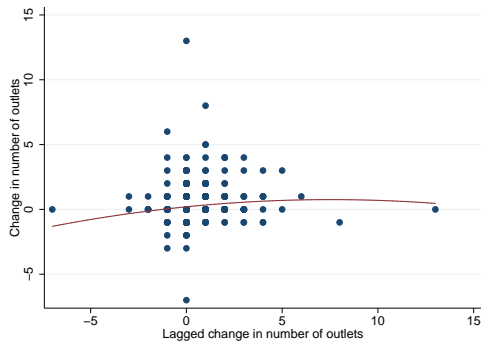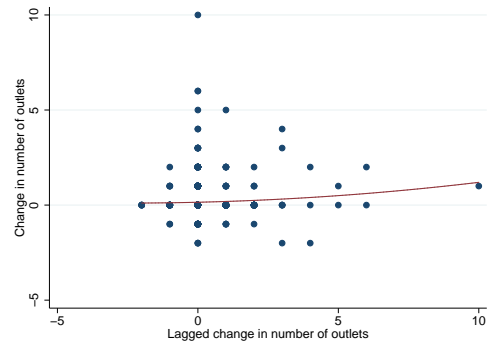
(b) Burger King
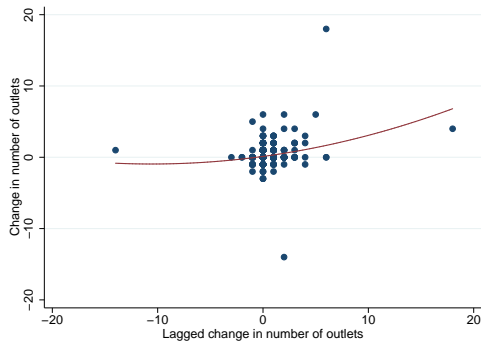
(c) Harvey's

(d) McDonald's

(e) Wendy's

Figure 2: Current Expansion and Contraction vs. Past Expansion and Contraction

positive relationship between past and current expansion for Harvey's, McDonald's, and Wendy's.[14]

In summary, these descriptive patterns suggest that there may indeed be a dynamic link between past and current profitability, as exhibited through the correlation between past and current expansion, and that if there is indeed such a link it may be different across firms. This, in turn, suggests a link between firm size, profitability, and market dominance.

3   Model

We consider a model of $i = 1, \ldots, I$ forward-looking firms in a retail industry that make decisions about operating in market m in every time period t. For a given market m, at the beginning of each time period t, firm i must decide whether to expand or contract operations, or make no changes to the number of outlets, i.e., $n_{imt} \in \{-K, -(K-1), \ldots, -1, 0, 1, \ldots, K-1, K\}$. Based on this decision the total stock of active outlets, $N_{imt} \geq 0$, in city[15] m at time t for firm i evolves as

$$N_{imt} = N_{im,t-1} + n_{imt}.$$

This formulation of expansion and contraction choices includes entry and exit as special cases. Entry occurs when $N_{imt} > 0$ following $N_{im,t-1} = 0$ and exit occurs when $N_{imt} = 0$ following $N_{im,t-1} > 0$. Each forward-looking firm i, whether incumbent or potential entrant, makes a decision n_{imt} to maximize its expected discounted stream of profits for each market m in each period t. In the tradition of discrete choice models of entry and exit (e.g., Bresnahan and Reiss, 1991a, 1991b, Berry, 1992), we define the reduced form one-shot payoff function as

$$
\begin{aligned}
\Pi_i(n_{imt}, &n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X_{mt}^R, X_{mt}^C, Z_{imt}, \nu_{imt}) \\
&= R(n_{imt}, n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X_{mt}^R, Z_{imt}^R) \\
&\qquad - C(n_{imt}, n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X_{mt}^C, Z_{imt}^C) + \nu_{imt}, \quad (1)
\end{aligned}
$$

where R is a revenue function, C is a cost function, and ν_{imt} is a private payoff shock. Here, n_{-imt} is a vector of the number of outlets that i's rivals choose to open or close in city m at time t. Similarly, N_{-imt} is a vector of the total number of outlets i's rivals have in market m in period t.

---

[14]We also explored alternative mechanisms that could lead to the patterns observed in Figure 2 and Table 3. For example, the demand for fast food could be growing in a market in which case all chains would see an increase in the number of outlets over time. We thank an anonymous referee for suggesting that, to rule out this alternative mechanism we could investigate whether the current expansion of a particular chain is correlated with the past expansion of its rivals. We found no evidence of this and so we proceed with the model introduced below, which is based instead on spillovers within a chain across time.

[15]Since we define each market to be a city we use these terms interchangeably.

The revenue and cost shifters are X_{mt}^R and X_{mt}^C, respectively; for instance, population and income would be included in X_{mt}^R, while real estate costs would be in X_{mt}^C. The variables Z_{imt}^R and Z_{imt}^C are, respectively, the unobserved (to the researcher) revenue and cost components of profitability of the retailers. Furthermore, θ^R and θ^C are vectors of model parameters. Finally, ν_{imt} is a privately known i.i.d. profit shock (i.e., a structural error) drawn from a distribution $F(\nu_i \mid S_{imt})$ with support $\mathcal{V}_i \subset \mathbb{R}$, where S_{imt} is the payoff relevant state defined below.

Let $S_{imt} = (N_{im,t-1}, N_{-im,t-1}, X_{mt}^R, X_{mt}^C, Z_{mt})$ denote the current payoff-relevant state from the perspective of firm i in market m at time t. To be more precise, we specify the one-shot payoff function parametrically as follows:

$$\Pi_i(n_{imt}, n_{-imt}, S_{imt}, \nu_{imt}) = \left[\theta_1^R X_{mt}^R - \theta_1^C X_{mt}^C - \theta_2 \sum_{j \neq i} N_{jmt} + \theta_3 N_{imt} + \gamma Z_{imt} \right] \cdot 1\{N_{imt} > 0\}$$

$$- \psi_1 \cdot 1\{N_{im,t-1} = 0, n_{imt} > 0\} - \psi_2 \cdot 1\{n_{imt} > 0\} \cdot n_{imt}$$

$$- \psi_3 \cdot 1\{N_{im,t-1} > 0, n_{imt} < 0\} \cdot n_{imt} + \nu_{imt} \cdot 1\{N_{imt} > 0\}. \quad (2)$$

The one-shot payoff represents revenue net of costs. Revenue is a function of the size of the market, as determined by $\theta_1^R X_{mt}^R$. However, if retailers face competition, their revenue is reduced by $\theta_2 \sum_{j \neq i} N_{jmt}$. Each store brings in additional revenue totaling $\theta_3 N_{imt}$. In addition to the costs $\theta_2^R X_{mt}^C$, the retailers face an entry cost, ψ_1, per-outlet expansion cost, ψ_2, and per-outlet salvage or scrap value ψ_3. The salvage or scrap value is gross of any contraction costs for the firm if these exist, e.g., penalties for breaking a real estate rental lease, severance payments to workers, etc.

The dynamic link between firm size and profitability arises in the model through unobserved composite profitability, $Z_{imt} \equiv Z_{imt}^R + Z_{imt}^C$. This composite profitability follows a firm-specific autoregressive process which evolves according to

$$Z_{imt} = \mu_i + \delta_i Z_{im,t-1} + \beta_{i1} N_{im,t-1} + \beta_{i2} N_{im,t-1}^2 + \beta_{i3} N_{im,t-1}^3 + \eta_m + \epsilon_{imt}, \quad (3)$$

where $\epsilon_{imt} \sim \mathrm{N}(0, \omega_i^2)$ is i.i.d. Therefore, the parameters $(\mu_i, \delta_i, \beta_{i1}, \beta_{i2}, \beta_{i3}, \omega_i, \eta_m)$ fully characterize the evolution of firm i's unobserved profitability in market m, net of the i.i.d. shock ν_{imt}.

This unobserved profitability measure has three main components. The first component, with parameter δ_i, is the *persistence of profitability*, or in other words, the extent to which retailers retain their past success (loosely speaking, "institutional retention or memory"). The second component, the collective terms with coefficients β_{i1}, β_{i2}, and β_{i3}, accounts for the changes in profitability as the chain's size in a given city changes over time (i.e., *intertemporal size spillover*). This empirical

specification of persistence in profitability and inter-temporal size spillovers in the unobserved profitability component Z_{imt} is similar to the learning by doing process in Benkard (2000, 2004), who in turn builds on Argote, Beckman and Epple (1990).[16] Finally, there is the drift component consisting of a firm-specific fixed effect, μ_i, which represents the long run average profitability or *brand effect* for firm i, and a city specific fixed effect, η_m. Since this is an AR(1) process with a potentially non-zero drift, due to the recursive structure, these effects account for any market- or firm-specific differences in the growth of unobserved profitability over time. Finally, ϵ_{imt} is a normally distributed i.i.d. innovation to unobserved profitability with standard deviation ω_i.

This specification allows for heterogeneity across firms since the parameters are chain-specific. Therefore, this specification captures firm-market-specific unobserved heterogeneity that is potentially serially correlated. Incorporating this time-varying endogenous profitability in the model is a critical aspect when studying the link between firm size, profitability, and market dominance. Although, this component of firm-specific profitability is unobserved to the researcher, we assume that Z_{imt} is observed by all firms. However, the model allows for some elements of a firm's profitability to be private information, e.g., proprietary technology or proprietary processes for supply chain management or service operations, or manufacture of products. This information is incorporated in ν_{imt}, which is known to firm i when making a decision but is not known by firm i's rivals. Furthermore, this is the key difference in the structural interpretation of the two components of profitability that are unobserved (to the researcher).

In summary, the model's structural parameters can be represented as $\alpha = (\alpha_1, \ldots, \alpha_I)$, where

$$\alpha_i = (\theta_1^R, \theta_1^C, \theta_2, \theta_3, \psi_1, \psi_2, \psi_3, \gamma, \delta_i, \beta_{i1}, \beta_{i2}, \beta_{i3}, \omega_i, \eta_m).$$

The one-shot payoff functions Π_i depend implicitly on these parameters. Given an initial state S_{imt} at time t, the firm's expected present discounted profits, prior to the private shock ν_{imt} being realized, is

$$\mathbb{E}\left[\sum_{\tau=t}^{\infty} \rho^{\tau-t} \Pi_i(n_{im\tau}, n_{-im\tau}, S_{im\tau}, \nu_{im\tau}) \,\middle|\, S_{imt}\right], \tag{4}$$

where ρ is the discount factor, $0 \leq \rho < 1$. The firm's objective is to maximize the present discounted

---

[16]The learning by doing literature typically estimates the relationship between costs or amount of inputs and cumulative output (as proxy for past experience). Benkard (2000) estimates the relationship between labor input and lagged cumulative output (allowing for depreciation to measure organizational forgetting), current output, and output of a related product (to capture spillovers across products). Our specification of size spillovers and institutional retention is more reduced form than, e.g., Argote, Beckman and Epple (1990), since it cannot disentangle the underlying sources of these effects such as employee turnover etc. This is primarily because of the nature of our data set. Potentially with better data the sources of such spillovers could be estimated. On the other hand, we embed this specification in fully specified dynamic oligopoly model with strategic interaction among firms.

value of its profits at each time period t taking as given the equilibrium action profiles of other firms. The expectation here is taken over the firms' actions and private shocks in the current period as well as the future evolution of the state variables, private shocks, and actions of all firms.

We follow the literature in specifying a dynamic game of incomplete information and focusing on Markov perfect equilibria (MPE) in pure strategies (e.g., Ackerberg, Benkard, Berry, and Pakes, 2007; Bajari, Benkard and Levin, 2007). In order to define the MPE strategies for the game we employ the following notation. Recall that S_{imt} is the payoff-relevant state for firm i in market m at time t. Let $\mathcal{S}_i$ denote the state space containing all feasible values of S_{imt} for firm i and let $\mathcal{N}_i$ denote firm i's choice set (i.e., $n_{imt} \in \mathcal{N}_i$). Also, define $\mathcal{S}$ to be the collection of $\mathcal{S}_i$. For simplicity, we let S_{mt} denote the market state, defined as the collection of the state variables S_{imt} of all firms in market m at time t. Similarly, let ν_{mt} denote the collection of the i.i.d. private shocks ν_{imt} of all firms in market m at time t. A Markov strategy for firm i is a function $\sigma_i : \mathcal{S} \times \mathbb{R} \to \mathcal{N}_i$ mapping payoff-relevant state variables and private information to the set of possible actions. We denote a profile of Markov strategies by $\sigma = (\sigma_1, \ldots, \sigma_I)$.

The ex-ante value function $V_i(S_{mt}; \sigma)$ gives the expected present discounted value of profits obtained by firm i when players use strategies σ and the market state is S_{mt}. Dropping the market and time indices here, we define the ex-ante value function recursively as

$$V_i(S; \sigma) = \mathbb{E}\left[\Pi_i(\sigma(S, \nu), S_i, \nu_i) + \rho\mathbb{E}\left[V_i(S'; \sigma) \mid S, n = \sigma(S, \nu)\right] \,\Big|\, S\right].$$

Here, $\sigma(S, \nu)$ denotes the action profile $(\sigma_1(S, \nu_1), \ldots, \sigma_I(S, \nu_I))$ and n denotes the same profile represented as $(n_1, \ldots, n_I)$. The outer expectation is over current values of the private shocks, ν, and hence current actions of rivals, given S. The inner expectation is with respect to the state variable next period, S', conditional on the state S and the actions of all firms in the current period (n_i, n_{-i}).

A MPE is defined as a Markov strategy profile σ such that no firm i has an incentive to unilaterally deviate from its strategy σ_i while its rivals are playing according to their strategies in σ_{-i}. Thus, for any firm i there is no alternative Markov strategy $\tilde{\sigma}_i$ that yields higher expected discounted profits (in terms of $V_i(\cdot)$) than σ_i while its rivals are using their strategies in σ_{-i}. Formally, σ is an MPE if, for all firms i, all market states S, and for all alternative Markov strategies $\tilde{\sigma}_i$, the following condition holds:

$$V_i(S; \sigma_i, \sigma_{-i}) \geq V_i(S; \tilde{\sigma}_i, \sigma_{-i}). \tag{5}$$

Note that for the alternative Markov strategy profile $(\tilde{\sigma}_i, \sigma_{-i})$, when the realized private information is ν, the realized actions are $\tilde{n}_i = \tilde{\sigma}_i(S, \nu_i)$ and $n_{-i} = \sigma_{-i}(S, \nu_{-i})$. Therefore, the recursive expression for the ex-ante value function is

$$
V_i(S; \tilde{\sigma}_i, \sigma_{-i}) = \mathbb{E}\Bigg[\Pi_i(\tilde{\sigma}_i(S, \nu_i), \sigma_{-i}(S, \nu_{-i}), S_i, \nu_i)
$$
$$
+ \rho\mathbb{E}\left[V_i(S'; \tilde{\sigma}_i, \sigma_{-i}) \mid S, \tilde{n}_i = \tilde{\sigma}_i(S, \nu_i), n_{-i} = \sigma_{-i}(S, \nu_{-i})\right] \Bigg| S\Bigg].
$$

Inside the outer expectation, the first argument of the payoff function Π_i is $\tilde{n}_i = \tilde{\sigma}_i(S, \nu_i)$, which is the implied action by firm i under strategy $\tilde{\sigma}_i$ when the state is S and the realized private information is ν_i. Similarly, the second argument is $n_{-i} = \sigma_{-i}(S, \nu_{-i})$, which is a profile of rival actions given the state S and the vector of private information shocks ν_{-i}.

We conclude the discussion of the theoretical model with a remark about equilibrium existence. Compared to discrete-state models, little is known about equilibrium existence in dynamic games with continuous states such as ours. The lack of theoretical results notwithstanding,[17] it might still be possible to explore the question numerically. It is computationally too burdensome to solve the full model numerically due to the large choice set and a prohibitively large state space including several discrete components and nine continuous components, five of which are firm-specific latent variables that are serially correlated and subject to feedback from lagged firm actions.[18] However, we successfully carried out an equilibrium search using a lower-dimensional counterpart to our empirical model.[19]

4   Estimation

Solving for even a single equilibrium of the game is both intractable analytically and prohibitively expensive computationally. Therefore, we employ a two-step approach to estimation which does

---

[17]Dutta and Sundaram (1998) provide results on existence of MPE in general Markovian games, however, it is not straightforward to verify their conditions for our empirical model since, which necessarily differs in certain ways from their theoretical framework.

[18]The state space has both discrete and continuous components. The discrete components are the number of outlets operated by each of the five firms along with indicators for smoking regulations and hosting the CFL Grey Cup. Using the summary statistics in Table 1 to determine the maximum number of outlets operated by each firm (and adding one value to indicate when firms are inactive), we note that there are over one billion possible combinations of these discrete components: $(50 + 1) \times (23 + 1) \times (54 + 1) \times (164 + 1) \times (23 + 1) \times 2 \times 2 = 1{,}066{,}348{,}800$. Additionally, there are nine continuous components: population, income, property value, minimum wage, and five firm-specific serially-correlated latent state variables.

[19]In particular, we extended the model of Igami and Yang (2015) by adding serially correlated, firm-specific states defined by $Z_{imt} = 0.5Z_{im,t-1} + 0.1N_{im,t-1} + \epsilon_{imt}$ in addition to population and income. We calibrated the remaining parameters using their estimates and successfully found an equilibrium using the Pakes and McGuire (1994) algorithm. We thank an anonymous referee for this suggestion.

not require one to explicitly solve the model and allows one to consistently estimate the model in the presence of multiple equilibria under standard assumptions. Our estimation procedure is an augmented version of the method proposed by Bajari, Benkard, and Levin (2007). In the first step, we estimate transition equations for the state variables, which characterize how the state variables evolve, along with the *reduced form* policy functions for each of the firms, which map state variables to actions and approximate the observed equilibrium behavior. In the second step, we use these quantities to simulate the ex-ante value functions which are in turn used to impose the equilibrium conditions in (5) via a minimum distance criterion function. This allows us to estimate the structural parameters without ever directly solving the model. As with the original method, we assume the state variables follow a first-order Markov process and that the data are generated by a single Markov perfect equilibrium and that all players expect the same equilibrium to be played in all periods. The key difference in our approach is the introduction of latent, firm-specific, and time-varying state variables to control for unobserved, but possibly persistent differences in profits due to inter-temporal size spillovers and persistence in profitability. To incorporate all of these features, we build on the sequential Monte Carlo approaches of Blevins (2015) and Gallant, Hong and Khwaja (2014, 2015).

4.1   First Stage Estimation

In the first stage we jointly estimate the posterior distributions of firm-specific latent state variables in each period (conditional on observed actions and states), parameters of the transition equations for the latent and observed states, and policy functions that condition on the levels of the latent states. We describe the estimation of state transition equations and policy functions in turn below, before turning to the second stage.

Let X_{mt}^R be the vector of state variables related to revenue in market m at time t, such as population, income and whether the city is hosting the CFL Grey Cup. Similarly, let X_{mt}^C be the vector state variables related to costs, such as property value and minimum wage. We summarize these exogenous state variables by collecting them in a vector $X_{mt} = (X_{mt}^R, X_{mt}^C)$. We use similar notation for the latent, endogenous state variables Z_{imt}. Let $Z_{mt} = (Z_{1mt}, \ldots, Z_{Imt})$ denote the vector of all firm-specific latent state variables in market m in period t. The variables Z_{imt} are endogenous. The evolution of these variables is influenced by the lagged actions of each firm as well as the lagged values $Z_{im,t-1}$, according to (3). As such, we estimate the parameters of the law of motion of Z_{imt} for each i jointly with the policy functions as described below. On the other hand,

the variables in X_{mt} are exogenous and so we estimate the parameters of the transition equations for these variables separately from the other parameters.

Collectively, let ϕ denote the vector of all first-stage parameters: the coefficients of the reduced form policy functions, the coefficients for the transition functions for the exogenous state variables, and the parameters $(\delta_i, \beta_{i1}, \beta_{i2}, \beta_{i3}, \omega_i)$ and η_m for the law of motion for Z_{imt} for each i and m as specified in (3).[20] The Bajari, Benkard and Levin (2007) method requires that in the first step we obtain consistent estimates of ϕ. Given data $\{n_{mt}, X_{mt}\}_{t=0}^T$ for the entire sample of $m = 1, \ldots, M$ markets, consistent estimates of ϕ can be obtained by maximizing the following likelihood function,

$$
\begin{aligned}
L_M(\phi) &= \prod_{m=1}^M \prod_{t=1}^T L_m(n_{mt}, X_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi) \\
&= \prod_{m=1}^M \prod_{t=1}^T \int l_m(n_{mt} \mid X_{mt}, Z_{mt}, \phi)\, p(X_{mt} \mid X_{m,t-1}, \phi)\, p(Z_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi)\, dZ_{mt}. \quad (6)
\end{aligned}
$$

Here, the M subscript indicates the dependence of the likelihood function on the entire sample, and the m subscript denotes dependence on the parameters for market m. The second equality follows from using the structure of the model to decompose the likelihood for the observed data in to three components after conditioning and integrating out the unobservable firm specific profitability components Z_{mt}. The three components in the likelihood are: (i) the firm-specific choice probabilities l_m conditional on Z_{mt} that represent the reduced form policy functions, (ii) the joint transition density of the observable state variables, and (iii) the posterior distribution of Z_{mt} given the data. It should be noted that although the posterior distribution of Z_{mt} is a reduced form component underlying it is the structural transition function for these unobserved state variables (3). It is estimated in the process of computing the posterior distribution as we describe below. We next discuss the specification and estimation of each component starting with the second and third components and then finally the reduced form policy function. We pay particular attention to the efficient calculation of the posterior distribution, which is the main technical innovation in our estimation approach.

We begin with the second component as it is the easiest to describe. The joint transition density of the observable state variables arises in this form because all variables in X_{mt} are exogenous and independent of both $Z_{m,t-1}$ and $N_{m,t-1}$. This allows us to estimate the parameters of the transition density of the observable state variables separately, whereas the parameters of the other

---

[20]Note that although the latent state transition parameters are structural parameters, they are being estimated in the first stage, so there is some overlap between the parameters in α (described in Section 3) and ϕ. Recall also that N_{mt} is a vector of the total number of outlets of firm i in market m at time t, and n_{mt} is the analog for firm choices.

two components of the likelihood L_M are estimated jointly. Specifically, the exogenous variables X_{mt} evolve according to a seemingly unrelated regressions (SUR) model described in the Appendix. Since major sporting events such as the CFL Grey Cup are announced to the public far ahead of time,[21] we assume that retailers have perfect foresight about whether a city will host the event in the future. A similar assumption is made for the roll-out of smoking by-laws, as these regulatory changes are often announced (and debated) well in advance of the enactment date (Lewis, 2012).[22]

The posterior distribution $p(Z_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi)$ is the third main component of the likelihood function L_M in (6) and is the implied distribution of Z_{mt} given the parameters and the data from last period. The main complication in estimating this model is that for each firm i, Z_{imt} is unobserved, serially correlated, and depends on lagged firm size through (3). In turn, the reduced form policy functions depend on this unobserved profitability state. Given this recursive relationship between the firm choices about stores, and hence size, and the the unobserved Z_{imt}, we estimate the reduced form policy functions jointly with the transition process for Z_{imt} by integrating with respect to the posterior distribution of Z_{imt} given the observed data.

Although, this posterior is not a model primitive, it can be calculated *recursively* using a sequential Monte Carlo or particle filtering procedure, that requires three pieces of information: (i) the initial distribution[23] of the unobserved Z_{mt}, (ii) the observation likelihood function l_m from (6) that relates the unobserved Z_{mt} to the observed n_{mt}, and (iii) the law of motion for the unobserved Z_{mt} given by (3). Once the process has been initialized using draws from the initial distribution, the recursive procedure for obtaining the posterior $p(Z_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi)$ can be described in two steps, starting with the lagged posterior $p(Z_{m,t-1} \mid N_{m,t-2}, X_{m,t-2}, \phi)$. First, the updating step applies Bayes' rule to obtain the *filtering* distribution given by (7) for period $t-1$,

$$p(Z_{m,t-1} \mid N_{m,t-1}, X_{m,t-1}, \phi) =$$
$$\frac{l_m(n_{m,t-1}, X_{m,t-1} \mid X_{m,t-2}, Z_{m,t-1}, \phi) p(Z_{m,t-1} \mid N_{m,t-2}, X_{m,t-2}, \phi)}{\int l_m(n_{m,t-1}, X_{m,t-1} \mid X_{m,t-2}, Z_{m,t-1}, \phi) p(Z_{m,t-1} \mid N_{m,t-2}, X_{m,t-2}, \phi)\, dZ_{m,t-1}}. \quad (7)$$

Second, the prediction step yields the *posterior* for Z_{mt} with period $t-1$ information by integrating with respect to the transition density for Z_{mt} given $Z_{m,t-1}$ and $N_{m,t-1}$ which we denote by q and

---

[21] For example, the cities hosting the 2013, 2014, and 2015 Grey Cup games were announced by the Canadian Football League 772, 626, and 613 days in advance, respectively.

[22] The assumption is invoked when forward simulating these market characteristics in estimating the second stage parameters as described below.

[23] The initial distribution $p(Z_{im,0})$ was taken to be N$(0,1)$ in the empirical application.

which is implied by (3):

$$p(Z_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi) =$$

$$\int q(Z_{mt} \mid Z_{m,t-1}, N_{m,t-1}, \phi) p(Z_{m,t-1} \mid N_{m,t-1}, X_{m,t-1}, \phi) \, dZ_{m,t-1}. \quad (8)$$

Thus, the parameters for the Z processes enter the likelihood through the transition density q.

The fundamental challenge is that this distribution is difficult to evaluate analytically because the firm's choices n_{imt} are determined by a complicated, non-linear best response function that depends on Z_{mt} based on a Markov Perfect Equilibrium of the dynamic game. This is seen in particular in (6) and (7) which include the observation likelihood l_m incorporating the reduced form policy function $l_m(n_{it} \mid X_{mt}, Z_{mt}, \phi)$. To solve this problem, we estimate the model using a particle filter approach based on Blevins (2015) by extending it to allow for endogenous feedback from the lagged size of firms. A non-linear particle filter makes it possible to approximate continuous distributions by a finite collection of weighted point masses. Thus, we replace the integral over the continuous distribution of Z_{mt} by a summation over a finite number of support points with weights. These weighted points are known as "particles" and are selected to incorporate all available information about Z_{mt} given the model, the data up to time t, and a vector of parameters to efficiently approximate the posterior distribution, which is then used to integrate the likelihood sequentially in every time period t. In this procedure, the particles are given by the draws from the distribution of Z_{mt} and the weights by the observation density l_m evaluated appropriately. More details about this algorithm are provided in the Appendix. We next describe the empirical specifications of the reduced form policy functions and the transition density of Z_{mt}.

There are two issues in estimating the reduced form policy functions. The first is that we do not know the true reduced form because finding it would involve finding the choice-specific value functions and projecting them onto exogenous state variables. In our application, firms choose each year how many stores by which to expand or contract, denoted by n_{imt}. Since the choices are naturally ordered and the costs are linear, in practice we approximate the reduced form policy functions by estimating an ordered probit specification where the latent index is a suitably flexible, linear-in-parameters function of the exogenous state variables X_{mt}, their interactions and squares, and the unobserved profitability Z_{mt}, and interactions between X_{mt} and Z_{mt}.[24]

---

[24]Bajari, Benkard and Levin (2007) suggest that the first stage reduced form policy functions should be estimated using flexible nonparametric methods such as kernel regressions or sieve estimators. However, in practice this is usually infeasible and the convention in the literature is to employ some form of parametric approximation, e.g., Ellickson and Misra (2008) use a multinomial logit.

Table 4: Grouped expansion and contraction decisions by chain

Change in Outlets	A&W	Burger King	Harvey's	McDonald's	Wendy's
$(..., -10]$	0	0	1	0	0
$(-10, -5]$	1	0	0	8	0
$(-5, -1]$	77	45	43	9	27
0	801	893	886	719	905
$[+1, +5)$	138	102	108	189	109
$[+5, +10)$	65	43	43	123	42
$[+10, ...)$	2	2	3	30	2

The second complication in estimating the first stage is that Z_{imt} is a serially correlated unobservable. To deal with this, we evaluate the integrated likelihood function of the ordered probit that approximates the reduced form policy function using draws from the particle filtering procedure described above (see (7) and (8)).

In particular, we estimate an ordered probit model over the choices $\mathcal{K} = \{k_1, k_2, \ldots, k_K\}$ with $k_1 < k_2 < \cdots < k_K$. These values may range from negative to positive, representing expansion and contraction decisions by firms. In our application, we choose $\mathcal{K} = \{-10, -5, -1, 0, 1, 5, 10\}$. We motivate this discretization with Table 4, which provides us the number of observations we see for each expansion or contraction decision. The table shows that there are many instances where the stock of outlets do not change, increase or decrease by one to ten outlets.

Each firm's decision depends on the value of a latent index, y^*_{imt}, which can be flexibly specified to depend on a vector W_{imt} of state variables and higher order interaction terms. In our most flexible specification, W_{imt} includes X_{mt}, the squares of the components of X_{mt}, the firm's own Z_{imt}, the average of rival Z_{-imt} values, and pairwise interactions between X_{mt} and the Z_{imt}, and rival Z_{-imt} values.[25] Following the literature, we use the following simple, but flexible linear specification for y^*_{imt} that includes higher-order terms and interactions:

$$y^*_{imt} = \phi' W_{imt} + \zeta_{imt},$$

where ζ_{imt} is an independent and normally distributed error term with mean zero and unit variance.[26] Also note that firm and city fixed effects are captured by Z_{imt}, as specified in (3), which is a component of W_{imt}.

---

[25] We omit certain interactions with indicator variables such as Grey Cup hosting, since including the square of such an indicator variable would introduce perfect multicollinearity. For the precise list of variables contained in W_{mt}, see the reduced form estimates in Table 13 in the Appendix.

[26] We normalize the variance of the error term to one because the coefficients in the payoff function are only identified up to scale. Moreover, since there is only a scalar unobservable there are no covariances to estimate or report. Finally, to normalize the scale of Z_{imt}, which is also included in the second stage, we fix $\phi^Z = 1$.

Decisions are related to the latent variable by a collection of threshold-crossing conditions:

$$
n_{imt} = \begin{cases}
k_1 & \text{if } y^*_{imt} \leq \vartheta_1, \\
k_2 & \text{if } \vartheta_1 \leq y^*_{imt} \leq \vartheta_2, \\
\vdots & \vdots \\
k_K & \text{if } \vartheta_{K-1} \leq y^*_{imt} \leq \vartheta_K.
\end{cases}
$$

The values $\vartheta_1, \ldots, \vartheta_K$ are the K cutoff parameters corresponding to each outcome. These cutoffs are estimated along with the index coefficients and the parameters in the law of motion for Z_{imt} using sieve maximum likelihood.

4.2 Second Stage Estimation

Once we have estimated policy functions that condition on Z_{imt} and parameters for the laws of motion of these variables, the second stage is conceptually identical to that of Bajari, Benkard, and Levin (2007). As such, we briefly summarize the steps below and reserve the full description for the Appendix. In effect, since we can simulate values of Z_{imt} we can treat these latent variables the same way as the observable variables for the purposes of the second stage.

Recall that we have estimated firm policy functions and state transition equations in hand from the first stage. Each firm's estimated policy function describes how it will act given a particular state and market structure. In equilibrium, each firm's strategy must agree with rival firms' beliefs. We assume that firms have rational expectations about state transitions, so firm beliefs and the state transition equations also agree. Therefore, the quantities estimated in the first stage can be used to simulate many sample paths, or alternative realizations, of the game in each market. Each such path is a sequence of firm actions and state transitions.

These simulated paths are used to estimate ex-ante value functions. We can then use the equilibrium conditions to form a minimum distance objective function to estimate the structural parameters of the model. Although the second stage is unmodified with respect to Bajari, Benkard, and Levin (2007), the incorporation of the latent state variables in the estimation in the first stage means that one can use the estimated structural parameters for the transition process of the latent states (3) to obtain best-response functions for firms that depend not only on the exogenous state variables, but also on the actions of the firm's rivals and the latent states, both of which are endogenous.

The second stage uses the estimates $\hat{\phi}$ to approximate the MPE policy profile σ. Let $\hat{\sigma}$ denote

the estimated policies corresponding to $\hat{\phi}$. Using these estimated policies, we can then generate the sequence of latent state vectors Z_t, and subsequently, the sequence of profits. Discounting and summing these profits yields an estimate of the valuations under the policies used.

Therefore, given policies $\hat{\sigma}$ and structural parameters α, we can simulate the *ex ante* value function for a particular firm i in any possible initial state $S_1 = (N_0, X_1, Z_1)$,

$$\bar{V}_i(S_1; \hat{\sigma}, \alpha) = \mathbb{E}\left[\sum_{\tau=1}^{\infty} \rho^{\tau-1} \Pi_i(\hat{\sigma}(S_\tau, \nu_\tau), S_\tau, \nu_{i\tau}; \alpha) \,\middle|\, S_1, \hat{\sigma}\right]$$

$$\simeq \frac{1}{\bar{S}} \sum_{s=1}^{\bar{S}} \sum_{\tau=1}^{T} \rho^{\tau-1} \Pi_i(\hat{\sigma}(S_\tau^s, \nu_\tau^s), S_\tau^s, \nu_{i\tau}^s; \alpha),$$

where we simulate $\bar{S}$ paths of length T. Variables with superscript s are for simulation s with $s = 1, \ldots, \bar{S}$. In particular, $\hat{\sigma}(S_\tau^s, \nu_\tau^s)$ denotes a vector of simulated expansion or contraction actions from the policy profile σ. The other variables are simulated according to their laws of motion, given the parameters α.

With this formulation, we can then repeat the same procedure using each of B different initial states, each under both the estimated optimal policies $\hat{\sigma}$ and when one firm uses a random alternative policy $\tilde{\sigma}_i$. Each alternative policy is generated by randomly perturbing the subvector of policy function parameters in ϕ by adding a random vector ϱ, where $\varrho \sim \mathrm{N}(0, \sigma_\varrho^2 I)$. Rather than simply additively perturbing the latent index used in the threshold crossing rules, these perturbations interact with the state variables. As discussed in Srisuma (2010), such alternative policies can contain additional information that is helpful in identifying the structural parameters of interest. We note that because the profit function is linear in parameters, as in Bajari, Benkard, and Levin (2007), many values needed in these simulations can be pre-computed and reused as α changes.

Under the true parameters α, by revealed preference, the estimated policy $\hat{\sigma}_i$ for firm i must yield a higher ex-ante valuation for that firm than any other policy $\tilde{\sigma}_i$, given that the other firms are using policies $\hat{\sigma}_{-i}$. Therefore, we can use the difference in these two ex-ante valuations as a basis for estimating α. Each initial condition and alternative policy yields an separate inequality.

Let b index the inequalities, with each inequality consisting of an initial market structure and state $S_1^b = (N_0^b, X_1^b, Z_1^b)$, an index for the unilaterally deviating firm i, and an alternative policy $\tilde{\sigma}_i$ for firm i. We denote the difference in valuations for firm i and inequality b by

$$g_b(\hat{\sigma}, \alpha) = \bar{V}_i(S_1^b; \hat{\sigma}, \alpha) - \bar{V}_i(S_1^b; \tilde{\sigma}_i, \hat{\sigma}_{-i}, \alpha). \tag{9}$$

In equilibrium, for the true parameter values, this difference should be positive based on a revealed preference argument. Hence, the minimum distance estimator of Bajari, Benkard, and Levin (2007)

chooses $\hat{\alpha}$ to minimize the violations of the equilibrium requirement in (9), i.e., minimizes the sum of squared deviations from positivity in the function $Q(\alpha)$ defined as follows,

$$Q(\alpha) = \frac{1}{B} \sum_{b=1}^{B} (\min\{g_b(\hat{\sigma}, \alpha), 0\})^2.$$

4.3 Identification

An important component of our model is the presence of serially correlated firm-specific unobserved profitability shocks Z_{imt} that arise due to size spillovers and persistence in profitability. These are integrated out of the likelihood in order to estimate and identify the structural payoff function. We use particle filtering or sequential Monte Carlo simulation to perform this integration. Particle filters being in essence a Bayesian procedure require some form of parametric assumptions.[27]

The fundamental intuition for identification is very similar to that for other models that rely on panel data in that we exploit variation in firm actions both within and across markets. However, since the model includes an unobservable serially correlated component Z_{imt}, this variation is used in conjunction with parametric distributional assumptions for the evolution of the unobservable component (see, e.g., Pakes, 1986). It is important to note that the parametric assumptions required to implement the particle filter procedure to integrate out the unobservable component Z_{imt} in estimation are also sufficient to identify the model parameters. Alternatively put, no extra parametric assumptions are required other than those needed to integrate out the serially persistent unobservable component, while recognizing that implementing a particle filter necessitates parametric assumptions given its Bayesian nature. Given this backdrop we next discuss identification in the heuristic style of Keane (2010) and Ching, Erdem and Keane (2013).[28]

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[27]Although, a particle filter is essentially a Bayesian procedure there is nothing that precludes it from being used as part of frequentist estimation routine as in our case (e.g., Chernozhukov and Hong, 2003). See e.g., Fernandez-Villaverde and Rubio-Ramirez (2007), Imai, Jain and Ching (2009), Norets (2009), and Fang and Kung (2010) for other applications of such Bayesian methods for integration. Hu and Shum (2012, 2013) provide conditions for nonparametric identification of dynamic models with serially correlated unobservables, however, they do not consider the case with endogenous feedback.

[28]As stated by Ching, Erdem and Keane (2013, p. 10) identification has multiple meanings: (1) "showing that the parameters of a model are identified *given the assumed model structure* (italics from authors). This may involve formal proof as well as intuitive discussion of what data patterns drive the estimates," (2) "analysis of which assumptions are *necessary* (italics from authors) to estimate a model or just convenient" (i.e., nonparametric identification analysis), and (3) examining "fragile identification," i.e., whether some "parameters may be formally identified but difficult to pinpoint in finite samples." Our discussion is in the spirit of (1) above. Regarding (2) they further state (p. 10, fn. 16), "Unfortunately, this literature has been misinterpreted by many researchers as suggesting that it may be possible to obtain 'model free evidence' about behavior. In fact, the approach of the nonparametric identification literature is to make a priori assumptions about certain parts of a model and then show that some other part (e.g., the functional form of utility or an error distribution) is identified without further assumptions." Moreover, they also state (p. 23), "It is important to remember that truly model-free evidence cannot exist. The simple empirical work

The main difference in our model from a typical dynamic oligopoly model for which the Bajari, Benkard and Levin (2007) procedure might be applicable is that in the first stage we jointly estimate the policy functions that condition on the latent profitability state Z_{imt}, along with the parameters of the transition equations (3) and the posterior distribution $p(Z_{imt} \mid N_{im,t-1}, X_{m,t-1}, \phi)$ for the firm specific latent profitability states. After the serially correlated unobservable states are integrated out, identification of the primitives in the one-shot payoff follows very much from the Bresnahan and Reiss (1991a) framework (see e.g., Tamer 2003). The distinction is that in our case the one-shot payoff includes the effect (γ) of the latent profitability shock Z_{imt}. A key exclusion restriction for identifying this effect is that the competitors' lagged firm size $N_{-i,m,t-1}$ affects the current period payoffs Π_{imt} in (2) but not the transition of the firm-i-specific unobserved profitability state Z_{imt} in (3). Alternatively, the exclusion restriction is that controlling for market fixed effects, there is no *direct* market wide profitability spillover of each firm's lagged size.[29] The key identification assumption is that a firm's lagged size only *directly* affects its own latent profitability state Z_{imt}. Of course, indirectly, in equilibrium through the actions of the firm, there is an effect on the actions and outcomes of its rivals. Additional excluded variables are the exogenous state variables X_{imt} that affect the current payoffs but not the evolution of the firm specific unobservable profitability component. Finally, in the forward simulation process, Z_{imt} can also be thought of as a pre-determined state variable that is excluded from the payoffs of the rival firms thus providing variation in payoffs across firms (see e.g., Pesendorfer and Schmidt-Dengler, 2003).[30]

The identification of institutional retention of past profits (δ_i) and intertemporal size spillovers ($\beta_{i1}, \beta_{i2}, \beta_{i3}$) in (3) may be considered in the following way. For firm i in market m at each time t, the particle filter starting with a draw from an initial distribution $p(Z_{im,0})$ uses (7) and (8) to recursively compute the sequence $(Z_{im,t}, Z_{im,t-1}, \ldots, Z_{im,0})$. Each $Z_{im,t}$ is projected on its lagged value ($Z_{im,t-1}$) and the corresponding lagged firm size ($N_{im,t-1}$). Using this projection the

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that promises to deliver such evidence always relies on some assumptions. These assumptions are often left implicit as a result of failure to present an explicit model. Often these implicit assumptions are (i) not obvious, (ii) hard to understand, and (iii) very strong."

[29]One could consider a more general model of market-wide spillovers from, say, R&D or advertising of each firm that makes the entire category more profitable over and above what the market fixed effect can capture. We abstract from that situation. One approach to do this could be to include the sum of the lagged sizes of all firms in (3). However, in that case one could not estimate heterogeneous firm specific spillovers. An alternative would be to include the vector $N_{m,t-1}$, with the firm-specific effects being potentially heterogeneous. This would make the model not only more computationally burdensome, but would also make identification more difficult by eliminating the exclusion restriction described above. Our current approach lies somewhere in between these two extremes. We allow for heterogeneity in spillovers across firms but restrict attention to internal firm specific spillovers.

[30]Recall, the assumption is that the profitability components $\{Z_{imt}\}_{i=1}^I$ are observed by all firms but are unobserved to the researcher. Thus, in the forward simulation process the firms know the draws of the $\{Z_{imt}\}_{i=1}^I$ from the particle filter when making choices about stores.

autocovariance between $Z_{im,t}$ and $Z_{im,t-1}$ provides a measure of the retention of profits for a chain. At the same time the variation over time within a market and across markets for a given chain in the predetermined lagged firm size helps to pin down the inter-temporal size spillover for that chain through the projection. As an analogy, Berry, Levinsohn and Pakes (1995, pp. 853-854), estimate unobserved marginal costs by computing the the residual of the inverted first order condition for the Bertrand-Nash equilibrium, and then projecting this on to a vector of product characteristics that account for costs. In our case we don't have first order conditions to invert since the firm's choice set is discrete but the Z_{imt} are computed recursively under the assumption that the firm is making optimal decisions about adding or subtracting stores. This optimality is incorporated through the observation likelihood l_m in (6) and (7).

As one example of the variation used to disentangle and identify the effects of lagged firm size $N_{im,t-1}$ and last period profitability $Z_{im,t-1}$ on current profitability Z_{imt}, suppose that the market characteristics $X_{m,t-1}$ in the last period increased so as to increase firm i's profit, and therefore $N_{im,t-1}$ has grown in response. Then suppose that X_{mt} in the current period returns to its previous value but we still observe the firm continuing to expand and increase N_{imt}. This can only be due to the positive size spillovers since the law of motion of Z_{imt}, i.e., (3), is independent of $X_{m,t-1}$ and so the given (fixed) level of serial correlation between $Z_{im,t-1}$ and Z_{imt} cannot explain the higher than usual growth in the firm size.[31]

Further basis for identification is provided by variation exhibited in the data for $\{N_{im,t-1}\}_{\forall i,m,t}$. The summary statistics displayed in Table 1 and discussed in Section 2 confirm that there is substantial variation in the number of outlets, ranging from 0 to as large as 164. Furthermore, variation in the observable market characteristics and demographics over time serve as important exclusion restrictions that have short- and long-term effects on the stock of outlets. The assumption here is that these characteristics move independently of a firms' unobserved profitability levels. For these exclusion restrictions to have identifying power, we need at least one exogenous variable that shifts the current and future shock of outlets. To ensure that we have such exogenous variables, we apply these identification arguments with data we have collected on the CFL Grey Cup hosting across cities, smoking regulation, and minimum wage policies across cities. For instance, cities tend to experience greater fast food expansion during the years in which they host the CFL Grey Cup versus years in which they do not. Furthermore, the fact that the CFL board of governors desires to rotate the event across all cities in a fair manner adds some randomness to the assignment

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[31]We thank an anonymous referee for suggesting this example.

of host cities. A similar argument can be made for the use of smoking regulation roll-out across municipalities over time, as this regulatory change is permanent once implemented, and will thus have an impact on the firms' future profitability gains from the firm size spillovers. Minimum wage regulation serves as another exclusion restriction; changes to the minimum wage have long-run cost implications for retail chain expansion decisions. Firms may expand at different rates in response to changes of the exogenous state variables, which may lead to additional expansion in future periods, which in turn aids in identification of the nature of persistence in profitability.[32]

5   Results

Table 5 contains estimates of the structural parameters of interest. This includes both the payoff parameters, estimated in the second stage, and the parameters of the law of motion of Z_{imt} for each firm, estimated in the first stage. We report bootstrap standard errors.[33] To avoid reporting very small numbers, coefficients on terms involving N are reported as coefficients on $N/100$, with corresponding scaling factors being used for variables involving N^2 and N^3.

In the first stage, we estimate an ordered probit via sieve maximum likelihood, including all exogenous variables and their interactions up to second order and interactions between exogenous variables and each firm's own Z_{imt} and the average value of Z_{jmt} for firm i's rivals, $j \neq i$. We report the remaining estimates, which are not of direct interest, in Tables 13 and 14 in the appendix. This includes the ordered probit coefficients and cutoffs as well as the market characteristic coefficients in the reduced form payoff function (2).

To prepare for the minimum distance estimator in the second stage, we generated $B = 3000$ random inequalities. Each inequality consists of an alternative policy function, which we generate by randomly perturbing the coefficients and cutoffs of the estimated reduced form policy function, and an initial state, which we draw randomly from the sample.

First, the coefficients on Z_{imt} in the payoff function are positive in both specifications that include it. The main parameters of interest are those related to the law of motion of Z_{imt}. Note that Z_{imt} is an AR(1) process with a drift term which also depends on the level, square, and cube of the lagged number of own outlets. The parameters of this process differ across firms. Recalling the law of motion for Z_{imt} as defined in (3), for each firm i in each market m we can think of increments to Z_{imt} being decomposed into three primary components as $Z_{imt} = \mu_{im} + \delta_i Z_{im,t-1} + \epsilon_{imt}$, where

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[32]We thank an anonymous referee for providing us with this insight.

[33]Given the computational burden we bootstrapped the standard errors using 96 replications (two replications per core on each of four 12-core machines) with replacement from the sample of 31 markets for 36 years.

Table 5: Structural Parameter Estimates

Parameter	Z (with spillovers)		Z (no spillovers)		No Z	
Z (γ)	0.0335	(0.0101)	0.0997	(0.0198)	–	–
Rival N (θ_2)	-0.0287	(0.1754)	-0.1880	(0.1325)	-0.2988	(0.3672)
Own N (θ_3)	1.0820	(0.2862)	0.5466	(0.3988)	0.5261	(1.0946)
Entry Cost (ψ_1)	-0.6208	(0.0754)	-0.5810	(0.0744)	0.2116	(0.2540)
Expansion Cost (ψ_2)	-0.2565	(0.0532)	-0.1568	(0.0717)	-0.4106	(0.1838)
Scrap Value (ψ_3)	0.0676	(0.0614)	-0.1924	(0.0920)	-0.2610	(0.1812)
A&W						
Firm f.e. (μ_{AW})	0.0000	–	0.0000	–	–	–
AR(1) (δ_{AW})	0.1050	(0.0024)	0.2090	(0.0062)	–	–
S.D. (ω_{AW})	1.0937	(0.0217)	0.4052	(0.0192)	–	–
N $(\beta_{\text{AW},1})$	0.0202	(0.0002)	–	–	–	–
N^2 $(\beta_{\text{AW},2})$	1.4061	(0.0185)	–	–	–	–
N^3 $(\beta_{\text{AW},3})$	-0.2949	(0.0022)	–	–	–	–
Burger King						
Firm f.e. (μ_{BK})	0.0411	(0.0003)	0.0125	(0.0003)	–	–
AR(1) (δ_{BK})	-0.0186	(0.0002)	0.0699	(0.0016)	–	–
S.D. (ω_{BK})	0.7549	(0.0164)	0.4052	(0.0192)	–	–
N $(\beta_{\text{BK},1})$	0.0590	(0.0005)	–	–	–	–
N^2 $(\beta_{\text{BK},2})$	0.3297	(0.0038)	–	–	–	–
N^3 $(\beta_{\text{BK},3})$	-0.0382	(0.0003)	–	–	–	–
Harvey's						
Firm f.e. (μ_{HARV})	0.0353	(0.0003)	0.0086	(0.0002)	–	–
AR(1) (δ_{HARV})	0.0097	(0.0001)	0.0986	(0.0040)	–	–
S.D. (ω_{HARV})	0.8447	(0.0193)	0.4052	(0.0192)	–	–
N $(\beta_{\text{HARV},1})$	0.0242	(0.0001)	–	–	–	–
N^2 $(\beta_{\text{HARV},2})$	1.5496	(0.0141)	–	–	–	–
N^3 $(\beta_{\text{HARV},3})$	-0.0050	(0.0001)	–	–	–	–
McDonald's						
Firm f.e. (μ_{MCD})	0.7076	(0.0157)	0.3612	(0.0077)	–	–
AR(1) (δ_{MCD})	0.2140	(0.0042)	0.3850	(0.0178)	–	–
S.D. (ω_{MCD})	0.6782	(0.0126)	0.4052	(0.0192)	–	–
N $(\beta_{\text{MCD},1})$	0.0335	(0.0005)	–	–	–	–
N^2 $(\beta_{\text{MCD},2})$	0.1277	(0.0011)	–	–	–	–
N^3 $(\beta_{\text{MCD},3})$	-0.1476	(0.0028)	–	–	–	–
Wendy's						
Firm f.e. (μ_{WEND})	0.1271	(0.0012)	-0.0032	(0.0001)	–	–
AR(1) (δ_{WEND})	0.1104	(0.0013)	0.2328	(0.0062)	–	–
S.D. (ω_{WEND})	0.5329	(0.0071)	0.4052	(0.0192)	–	–
N $(\beta_{\text{WEND},1})$	-0.0339	(0.0003)	–	–	–	–
N^2 $(\beta_{\text{WEND},2})$	3.2709	(0.0432)	–	–	–	–
N^3 $(\beta_{\text{WEND},3})$	-0.0191	(0.0002)	–	–	–	–
Market Char. (θ_1)	Yes		Yes		Yes	

μ_{im} is a firm-market-specific drift, δ_i is the autoregressive or persistence in profitability parameter, and ϵ_{imt} is an i.i.d. Gaussian innovation (with standard deviation ω_i). The drift component, $\mu_{im} = \mu_i + \eta_m + \beta_{i1} N_{im,t-1} + \beta_{i2} N_{im,t-1}^2 + \beta_{i3} N_{im,t-1}^3$, is further composed of three parts: a firm fixed effect or brand effect (μ_i), a market fixed effect (η_m), and a firm-specific inter-temporal size spillover component ($\beta_{i1}, \beta_{i2}, \beta_{i3}$). The main specification, "Z (Spillovers)", contains all of these components, the second specification, "Z (No Spillovers)", omits the size spillover component, and the specification with no persistence, "No Z" omits the Z_{imt} process entirely.[34]

The estimates indicate that there is substantial heterogeneity across chains. Since the drift terms are similar to firm fixed effects, we normalize μ_{AW} to be zero and estimate the relative differences for the other firms. In the main specification with spillovers, we expected the coefficient on quadratic N term to be positive and the cubic coefficient to be negative, reflecting initially increasing, then decreasing returns in the increments to the latent payoff as a function of the lagged number of stores.[35] As shown in Table 5, this is indeed the case for all firms.

Overall, the brand or firm fixed effect and persistence in profits (autoregressive coefficient) is strongest for McDonald's, which also has nearly the smallest standard deviation (second only to Wendy's). Among all five retailers, the evolution of A&W's latent state has the smallest brand effect and also the largest standard deviation. Burger King then has the second smallest brand effect term, second highest standard deviation, and relatively weak size spillovers. Compared to the other firms, Harvey's and Wendy's have the largest quadratic spillover coefficients and moderately large brand effects and autoregressive coefficients. This indicates a more transient payoff benefit from having built more outlets in previous periods when compared, for example, to McDonald's.

The retailers appear to be sensitive to competition, but not in a statistically significant way, and earn higher profits as they build additional outlets. The insignificant competitive effect indicates that consumers may view these chains as being relatively differentiated. Alternatively, this could also arise if the density of locations in each city is relative low so that the competition between locations is small. Among the cost estimates, for the main specification the estimated initial cost of entry (ψ_1) is more than twice the cost of building a single store (ψ_2). The estimated scrap values (ψ_3) are not significantly positive, indicating that liquidating outlets is not lucrative.

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[34]The "No Z" specification does, however, include firm and city fixed effects which serve a similar role as the drift terms in the Z processes. We do not, however, report the estimates of these fixed effects as drift terms in Table 5 to avoid confusion, since the values are not directly comparable across specifications.

[35]We note that this is not the same as thinking about returns to scale more broadly, for which one has to consider the entire payoff function and the dynamic aspects of the problem including the entry and expansion costs and competitive effects.
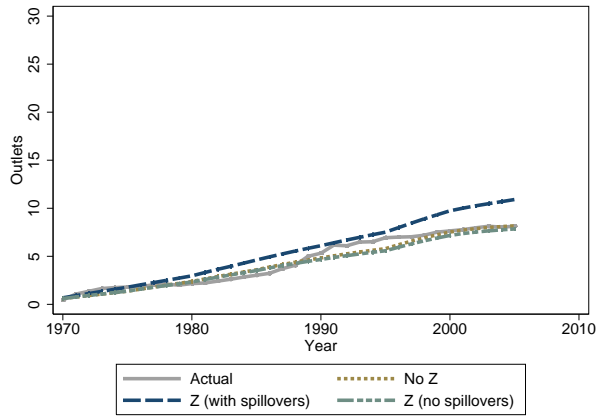
5.1 Model Fit Comparison

Having estimated our model with three different specifications, we now seek to determine which specification best fits the observed data. We use three statistical criteria to evaluate the model fit: the Akaike information criterion (AIC), the Bayesian information criterion (BIC), and the mean squared error (MSE) of the simulated model predictions.

The AIC and BIC criteria take into account the trade-off between the number of parameters and the relative fit (with the penalty being larger for BIC than AIC). The AIC and BIC values for each model are reported in Table 13 in the appendix. In terms of both AIC and BIC, the "Z (Spillovers)" specification is clearly preferred (AIC = 8018.45 and BIC = 8159.03) followed by the "No Z" and "Z (No Spillovers)" specifications, which have larger and quite similar AIC and BIC values (respectively, AIC = 8232.80 and BIC = 8320.85; AIC = 8233.10 and BIC = 8344.33).

Next, we carry out simulations using each of the estimated specifications and plot the average number of outlets predicted by each. We then compare the fit by evaluating the mean squared error in the simulated predictions. To implement the model simulations and counterfactuals, we employ a similar forward simulation approach as in Benkard, Bodoh-Creed, and Lazarev (2010), which does not require one to solve a computationally intractable dynamic model. We provide additional technical details about the simulations that follow in the appendix.

The main findings from our model fit simulations are displayed in Figures 3 and 4. Figure 3 plots the evolution of store counts over time across these different scenarios, with the actual dynamics found in the data serving as a benchmark. For each firm, we plot the average number of outlets (averaged across markets) each period observed in the data and the average simulated number of outlets with each of three model specifications (averaged over 250 simulations for each market, then across markets). The simulation runs are initialized using the observed market characteristics and number of outlets at the beginning of our sample. Since we are essentially forecasting 36 years ahead, differences relative to the observed number of outlets are expected. However, in that sense all three models perform quite well.

The "Z (Spillovers)" specification, which includes persistent unobserved profitability via the Z process (3) and allows for size spillovers, also has the lowest MSE. The ranking of the other two specifications is interchanged under the MSE criterion, with the "No Z" specification without the Z process having the highest MSE. Additionally, the estimated entry cost for the "No Z" specification is positive, as shown in Table 5, which is contrary to economic theory. This underscores the need

(a) A&W
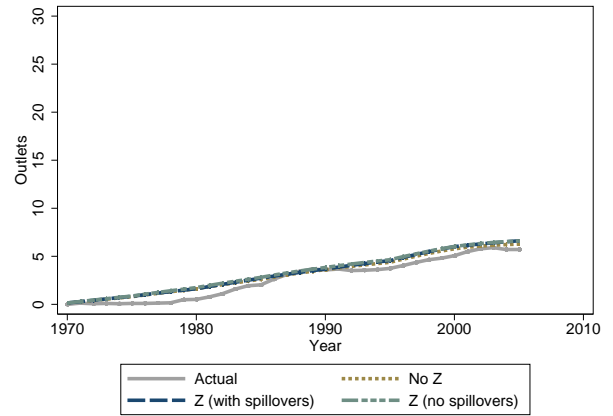
(b) Burger King
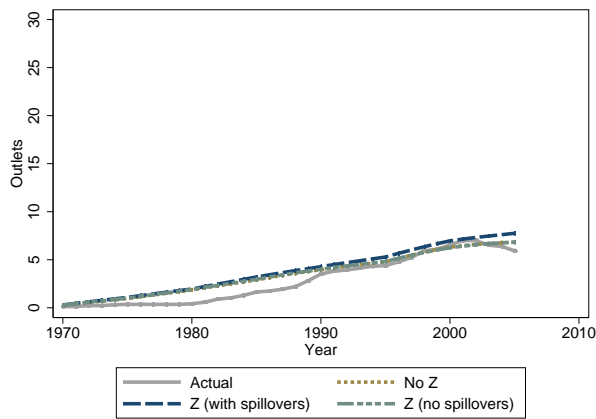
(c) Harvey's

(d) McDonald's

(e) Wendy's

Figure 3: Comparison of Average Number of Outlets by Model Specification

Figure 4: Mean Squared Error of Simulated Model Fit

for incorporating firm specific unobserved heterogeneity and the choice of "Z (Spillovers)" as our preferred specification.

5.2 Drivers of McDonald's Dominance

With the estimated structural model of retail chain dynamics, we can better understand the evolution of market dominance based on firm specific heterogeneity in serially correlated unobserved profitability as determined by combinat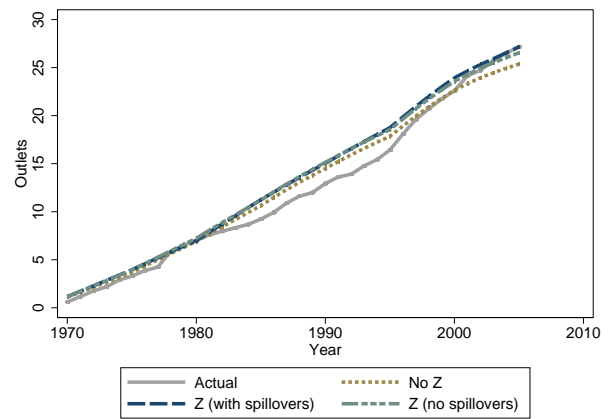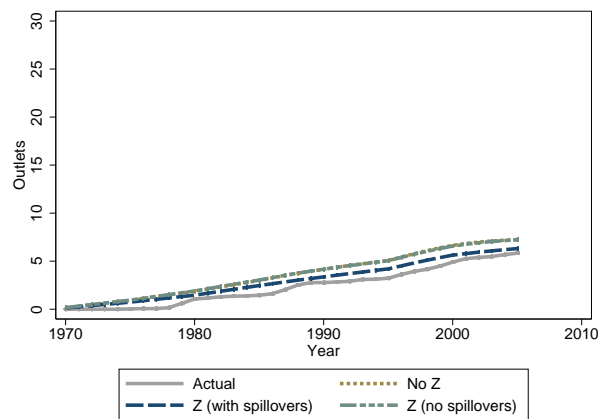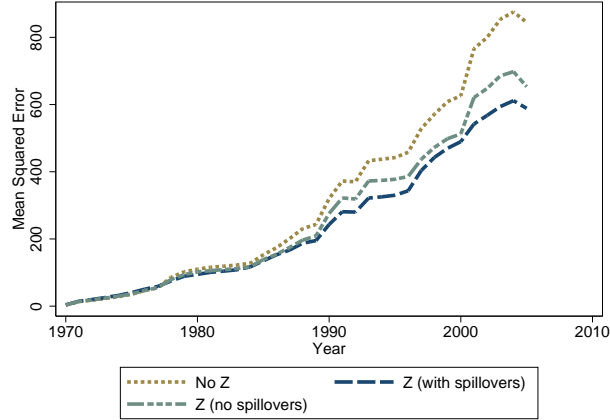ion of brand effects (μ_i), inter-temporal size spillovers ($\beta_{i1}, \beta_{i2}, \beta_{i3}$) and persistence in profitability (δ_i). First, as described above McDonald's has the largest value of the brand effect (μ_i) which is more than five times that of any of the other chains. Note, also that A&W's brand effect is normalized to zero for identification. Since the brand effect is a time invariant fixed effect it permanently raises McDonald's profitability relative to its competitors. Second, McDonald's also has a very high persistence parameter (δ_i), almost twice as large as any of its rivals. Given this high degree of serial persistence in unobserved profitability for McDonald's, the cumulative effects of its brand strength are magnified much more than the one-period competitive advantage that the brand effects seems to suggest. Third, the inter-temporal size spillover parameters ($\beta_{i1}, \beta_{i2}, \beta_{i3}$) for McDonald's have the expected signs, although, these are not particularly larger compared to those of its rivals. However, once again their effect is magnified in combination with the strong persistence in profitability to significantly increase McDonald's long run cumulative profitability. On the other hand, while the competitors of McDonald's may have somewhat stronger inter-temporal size spillovers they also exhibit much weaker persistence in profitability. This implies that cumulative benefit from having built more outlets in previous

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Table 6: Means and Variances of Stationary Distributions of Unobserved Profitability

	Mean	Variance
A&W	0.1028	1.2096
Burger King	0.1356	0.5701
Harvey's	0.1285	0.7136
McDonald's	1.0172	0.4821
Wendy's	0.2462	0.2875

periods when compared, for example, to McDonald's is much more transient for its rivals.

Given the important role played by the persistence in profitability we next investigate and compare the unobserved profitability processes for the different retail chains in terms of the statistical properties of the processes and their stationary distributions.

First, Table 6 reports the means and variances of the stationary distributions for each firm. Since the mean values vary across markets, due to the inclusion of market-specific drift parameters, we report averages across all markets. The mean for McDonald's is over four times larger than that of Wendy's, which has the second highest mean, and over seven times larger than the other chains. Furthermore, McDonald's also has the second smallest variance, which is about 68% larger than that of Wendy's, which has the smallest variance. A&W has both the smallest mean, just behind Harvey's and Burger King, and the largest variance, being 70% larger than that of Harvey's (the next highest) and four times as large as Wendy's (the smallest).

As discussed above, the estimated inter-temporal spillover coefficients $(\beta_{i1}, \beta_{i2}, \beta_{i3})$ imply the biggest effects for A&W, Wendy's and Harvey's. However, these coefficients provide a short-run measure of spillovers from the one-period lag stock of stores on current profit. Given serial persistence in profits it is crucial to also consider the long-term spillovers that account for the cumulative effects of these short-run size spillovers based on persistence in profitability. In order to assess the long-term effects of spillovers due to persistence we conduct the following exercise. We compare the autocovariance functions of Z_{it} for each firm i. For k periods ahead, the autocovariance for firm i is $\text{Cov}(Z_{it}, Z_{i,t+k}) = \delta_i^k \omega_i^2 / (1 - \delta_i^2)$. Therefore, the persistence in the Z_{it} process is determined by both the autocorrelation coefficient, δ_i, and the standard deviation of the i.i.d. innovations, ω_i. Larger values of either parameter will tend to increase the time until the process reverts back to the mean following a shock implying greater persistence in profitability. Table 7 reports the autocovariances for $k = 1, 2, 3, 4$ periods ahead for each firm i. McDonald's has by far the most persistence, due largely to its large autoregressive parameter. For A&W the initial effect of a shock

Table 7: Autocovariances of Unobserved Profitability Processes by Firm

	1	2	3	4
A&W	0.1270	0.0133	0.0014	0.0001
Burger King	0.0106	0.0002	0.0000	0.0000
Harvey's	0.0069	0.0001	0.0000	0.0000
McDonald's	0.1031	0.0221	0.0047	0.0010
Wendy's	0.0317	0.0035	0.0004	0.0000

is larger than for McDonald's, due to the large variance parameter for A&W, but the effect decays more quickly than for McDonald's.

This analysis illustrates the drivers of McDonald's dominance. It shows that even if the absolute short-run levels of inter-temporal spillovers are relatively small, "institutional memory" or retention of these spillovers over a long enough period of time can lead to drastic cumulative effects. Overall, what our analysis reveals is that brand effects, inter-temporal size spillovers and persistence in profitability can interact to have critical long-term consequences even if their short-run effects are measurably small. Thus, even with small transitory effects these three sources of profitability can still combine to have far reaching repercussions that can affect a firm's market dominance (or lack thereof) and market structure.

5.3   Robustness of McDonald's Dominance

Our next set of counterfactual simulations evaluates the robustness of McDonald's dominance in light of shocks handicapping it relative to its competitors, and demand and supply shocks to the economy.

First, we test the strength of McDonald's dominance relative to its rivals' capabilities by imposing an initial handicap on McDonald's in two ways. We summarize the results in terms of discounted counterfactual profit shares (i.e., profit shares based on the present discounted value of profits for each firm as opposed to the period-by-period profits). First, in Table 8 we posit a series of scenarios in which all of the rival firms competing with McDonald's are endowed with between 0 to 10 more outlets than they actually had in the first time period. These simulations indicate that even if the rival firms were endowed with one to two additional outlets each in the first year, McDonald's would still capture nearly one fifth of discounted profits. Next, in Table 9 we consider scenarios where the rivals have initial draws of their unobserved profitability, Z_i, taken from the stationary distribution for McDonald's instead of their own and then additionally inflating

Table 8: Discounted Profit Shares When McDonald's Rivals Have Additional Outlets

Additional Rival Outlets	0	1	2	3	4	5	6	7	8	9	10
A&W	0.0451	0.0364	0.0372	0.0383	0.0398	0.0411	0.0421	0.0430	0.0437	0.0444	0.0449
Burger King	0.2051	0.2487	0.2560	0.2581	0.2601	0.2602	0.2621	0.2643	0.2663	0.2680	0.2695
Harvey's	0.2244	0.2472	0.2518	0.2530	0.2546	0.2560	0.2573	0.2596	0.2601	0.2622	0.2627
McDonald's	0.3142	0.2060	0.1937	0.1872	0.1810	0.1761	0.1710	0.1653	0.1597	0.1533	0.1473
Wendy's	0.2113	0.2617	0.2614	0.2633	0.2645	0.2666	0.2675	0.2677	0.2701	0.2722	0.2756
HHI	0.2378	0.2352	0.2361	0.2365	0.2368	0.2370	0.2375	0.2381	0.2390	0.2401	0.2413

Table 9: Discounted Profit Shares When McDonald's Rivals Have Better Initial Draws of Unobserved Profitability

Initial Draw Increase	0%	100%	200%	300%	400%	500%	1000%	2000%	3000%
A&W	0.0451	0.0368	0.0322	0.0299	0.0285	0.0279	0.0279	0.0300	0.0328
Burger King	0.2051	0.2316	0.2412	0.2468	0.2499	0.2522	0.2551	0.2576	0.2599
Harvey's	0.2244	0.2381	0.2468	0.2508	0.2531	0.2555	0.2592	0.2600	0.2616
McDonald's	0.3142	0.2589	0.2384	0.2286	0.2218	0.2165	0.2081	0.2025	0.1929
Wendy's	0.2113	0.2345	0.2413	0.2439	0.2466	0.2480	0.2497	0.2499	0.2528
HHI	0.2378	0.2337	0.2352	0.2364	0.2374	0.2380	0.2387	0.2383	0.2382

the initial draws by percentages ranging from 0% (no inflation) to 3000%. Both of these exercises illustrate that rival firms would need some initial advantage in order to compete on a level playing field with McDonald's.

In our next simulations, we look at how well McDonald's dominance withstands economic downturns through both demand and supply shocks. In the first four columns of Table 10, we simulate the impact of a sudden drop in demand through a fall in income in 2006, around the time of a major economic downturn in North America. The first case captures the event in which income in 2006 drops 10% from the 2005 level, the second case captures 5% drop relative to the 2005 level, the third case captures 5% increase from the 2005 level, and the fourth case captures a 10% increase from the 2005 level. Notice that McDonald's actually gains in terms of profit share in response to both positive and negative shocks.[36] For negative shocks, these profit share gains come largely at the expense of Harvey's and Wendy's and for positive shocks, largely at the expense of Burger King and Wendy's. In a similar manner, the last four columns of Table 10 report the simulated results of a supply side cost shock in the form of changes to minimum wage. We consider the effect of sudden increases or decreases in wage by 5 or 10 percent in 2006 on discounted profit shares. As in the

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[36]We note that the *share* of McDonald's profit can increase under this scenario even while profits in *levels* decline. Recall that we define profit share as the share of the present discounted valuations, but since we do not observe market-level profits we cannot simulate counterfactual profits in monetary terms.

Table 10: Discounted Profit Shares in Response to Changes in Economic Conditions

| | Change in Income | | | | Change in Minimum Wage | | | |
	-10%	-5%	5%	10%	-10%	-5%	5%	10%
A&W	0.0451	0.0457	0.0475	0.0482	0.0461	0.0463	0.0469	0.0471
Burger King	0.2021	0.2003	0.1978	0.1977	0.2001	0.1991	0.1992	0.2002
Harvey's	0.2176	0.2188	0.2170	0.2155	0.2166	0.2186	0.2173	0.2162
McDonald's	0.3266	0.3262	0.3331	0.3349	0.3312	0.3285	0.3305	0.3298
Wendy's	0.2087	0.2089	0.2047	0.2037	0.2060	0.2076	0.2061	0.2067
HHI	0.2404	0.2402	0.2413	0.2415	0.2412	0.2405	0.2408	0.2405

previous counterfactual, McDonald's remains the market leader following downward and upward shocks to the minimum wage. Thus, major economic shocks would not appear to affect McDonald's overall leadership position in the retail hamburger industry. In summary, we find that McDonald's dominance is very robust to shocks to demand, supply, and the capabilities of its competitors.

6 Conclusions

Our paper presents a new empirical model of retail chain dynamics that allows for endogenous firm size, heterogeneous effects of size on future profitability, and the consequences for market dominance and evolution. Through a firm specific unobservable the model accounts for a heterogeneous dynamic link between firm size and profitability, that may arise from inter-temporal size spillovers and persistence in profitability. The dynamic game is estimated by using a particle filter based method to extend the Bajari, Benkard and Levin (2007) two-step estimator to incorporate time varying firm specific unobserved heterogeneity subject to endogenous feedback.

Using data on Canadian hamburger retail chains, the estimated model reveals a link between current size, future profitability and market dominance. The analysis accounts for the possibility that a forward-looking firm will incorporate such size spillovers in its decision to expand or contract, which has implications for estimating the effect of a firm's size on its payoffs and decisions. The estimated model produces several insights. First, it provides evidence of heterogeneity in brand effects, inter-temporal size spillovers and persistence in profitability across firms in a setting where firms interact in a strategic manner. Second, the estimated model shows that McDonald's dominance in Canada can be attributed to such effects. Third, it also shows that McDonald's advantage via this dynamic linkage is robust to hypothetical scenarios with unexpected demand and supply shocks, and when McDonald's faces a competitive handicap resulting from arbitrary increases in rival outlets or profitability during the initial year. Overall, we find that the heterogeneous dynamic

linkage between firm size and profitability shows how some firms become dominant and others falter as they expand, thus affecting market structure. Studying this relationship is an empirical matter that is valuable not just because of strategic considerations for firms when deciding on expansion or contraction but also for public policy, i.e., whether some firms can become dominant and marginalize others, and the consequences for market structure. Finally, we also find that our baseline model that incorporates a serially correlated unobserved profitability component fits the data better than alternative models that ignore such firm specific latent heterogeneity.

Our work extends the basic entry and exit framework to allow for expansion and contraction which may have broader application in other contexts where endogenous firm size or firm specific time varying unobserved heterogeneity is important. More specifically, our model and estimation framework could be applied to other retail industries in which key decisions revolve around expansion and contraction via stores. We believe that our framework may uncover similar firm size spillovers and persistence in profitability in settings in which other studies have demonstrated a growing wedge between large and small enterprises (e.g., Jia, 2008, Basker, Klimek, and Van, 2012).

Lastly, it is beyond the scope of our paper to identify and examine the underlying mechanisms and specific elements of firm capabilities that lead to a dynamic link between firm size, profitability market dominance. This could be a topic for future research. Another caveat is that we abstract away from potential national level expansion strategies in our analysis. For example, Holmes (2011) examines Walmart's expansion based on its network of distribution centers and the economics of density but in a single agent framework that does not account for strategic interaction between firms. Such decisions may be important to consider if the retail chains are concerned about geographic risk (e.g., Aguirregabiria, Clark, and Wang, 2013). However, there are two reasons we believe such a concern may be mitigated in our context. First, hamburger retail store expansion or contraction decisions are almost always made at the level of city headquarters. Second, although, a likely strategy borne out of risk aversion may involve diversification of outlets across cities this would be counteracted by an incentive to avoid losing the potential benefits of city-wide firm size spillovers. Finally, estimating expansion as a retail network decision is currently infeasible, in the form of a fully dynamic game with a rich state space, heterogeneous players, and serial correlation in unobservables with endogenous feedback. On the other hand, this suggests a very challenging but ambitious avenue for future research.

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A Web Appendix (Not for Publication)

A.1 First Stage Estimation

In the first stage, we maximize the log likelihood function with respect to the first-stage parameters, denoted ϕ. For each value of ϕ, the algorithm for evaluating the log likelihood function is as follows:

1. For each market $m = 1, \ldots, M$:

   (a) Draw R particles, denoted $\tilde{Z}^r_{m,0}$, from the initial distribution of firm-specific spillover levels. Each draw $\tilde{Z}^r_{m,0}$ is a vector of length I, with one component for each firm.

   (b) For each $t = 1, \ldots, T$:

      i. Transition each of the R particles $\tilde{Z}^r_{m,t-1}$ from period $t-1$ through the joint transition equation as determined by (3), given the current value of ϕ, to obtain a new collection of particles denoted[37] Z^r_{mt}.

      ii. Calculate and store the joint likelihood (the product of the firm-specific choice probabilities) associated with each particle, given the values of n_{mt} and X_{mt} from the data. Denote these values by p^r_{mt} for $r = 1, \ldots, R$.

      iii. Calculate and store the log likelihood value, given by log of the averaged joint particle-specific likelihood values: $l_{mt} = \ln\left(\frac{1}{R}\sum_{r=1}^{R} p^r_{mt}\right)$.

      iv. Assign the likelihood p^r_{mt} as an importance weight for each particle Z^r_{mt}.

      v. Draw R new particles, denoted $\tilde{Z}^r_{mt}$ for $r = 1, \ldots, R$ by sampling with replacement from the particles $\{Z^r_{mt}\}^R_{r=1}$ in proportion to the assigned importance weights.[38]

2. Return the value of the log-likelihood function for the trial value of ϕ, $L_M(\phi) = \sum_{m=1}^{M}\sum_{t=1}^{T} l_{mt}$.

A.2 Second Stage Estimation

In the second stage, we choose structural parameters α in order to maximize the minimum distance objective function $Q(\alpha)$. Initially, we construct a collection of B "inequalities" as follows:

1. Randomly draw B initial market structures, denoted (N^b_1, X^b_1, Z^b_1) for $b = 1, \ldots, B$, consisting of exogenous state variables, firm-specific observable variables, and firm-specific unobservable spillover levels.

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[37] The distinction between $\tilde{Z}^r_{mt}$ and Z^r_{mt} is intentional. The former are draws from the period $t-1$ filtering distribution (for the latent states at time $t-1$ given period $t-1$ information) while the latter are draws from the period t prediction distribution (for the latent states at time t using period $t-1$ information).

[38] Again, the tilde denotes that these are draws for period t updated with period t information.

2. For each initial market structure indexed by $b = 1, \ldots, B$, choose a single firm i and draw a random alternative policy function for that firm by adding a random vector $\varrho^b \sim N(0, \sigma_\varrho^2 I)$ to the subvector of parameters in $\hat{\phi}$ related to the first-stage policy functions. Let $\tilde{\sigma}^b$ denote the policy profile where firm i is using the alternative policy while all other firms use the estimated policies from $\hat{\sigma}$.

Once the inequalities are determined, calculate the objective function $Q(\alpha)$ for any α as follows:

1. Given the estimated first-stage parameters $\hat{\phi}$ and a vector of structural parameters α, repeat the following steps for each of the initial market structures indexed by $b = 1, \ldots, B$:

    (a) Draw S sample paths of length T, each starting at (N_1^b, X_1^b, Z_1^b), using the laws of motion determined by the given parameters $\hat{\phi}$ and α. Store the discounted profits for the chosen firm i for the simulated path.

    Specifically, for each simulated path $s = 1, \ldots, S$, for each time $t = 1, \ldots, T$:

        i. Using the law of motion estimated in the first stage, draw shocks to simulate a new vector of firm-specific spillover levels, $Z_t^{b,s}$.

        ii. Using the fitted SUR model, draw shocks and simulate new values for each of the exogenous variables, $X_t^{b,s}$.

        iii. Using the new spillover levels and exogenous variables, draw structural shocks and evaluate the estimated policies $\hat{\sigma}$ in order to simulate each firm's expansion or contraction decision, $n_{it}^{b,s}$.

        iv. Calculate the stock of outlets for each firm, $N_{it}^{b,s} = N_{i,t-1}^{b,s} + n_{it}^{b,s}$.

        v. Calculate the structural period profits for firm i in period t under $\hat{\sigma}$, denoted $\hat{\pi}_{it}^{b,s}$:

    $$\hat{\pi}_{it}^{b,s} = \Pi(N_{it}^{b,s}, N_{-it}^{b,s}, X_t^{b,s}, Z_t^{b,s}, \zeta_t^{b,s}, \alpha).$$

    (b) Calculate the discounted profits for firm i from the perspective of the initial state for each simulated path, $V^{b,s}(\hat{\sigma}, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \hat{\pi}_{it}^{b,s}$.

    (c) Estimate the ex-ante value of being in state (N_1^b, X_1^b, Z_1^b) for firm i (when firms use the estimated policies in the profile $\hat{\sigma}$) by averaging the discounted profits over all S paths: $\bar{V}^b(\hat{\sigma}, \alpha) = \frac{1}{S} \sum_{s=1}^{S} \hat{V}^{b,s}(\hat{\sigma}, \alpha)$.

    (d) Repeat steps in part 1a to simulate S alternative paths of length T, also starting at (N_1^b, X_1^b, Z_1^b) and using the parameters $\hat{\phi}$ and α, but using the alternative policy from $\tilde{\sigma}^b$ for firm i. Let $\tilde{\pi}_{it}^{b,s}$ denote the profits earned by firm i in period t along path s.

	Population$_t$		Income$_t$		Property Value$_t$		Minimum Wage$_t$	
Population$_{t-1}$	1.0043	(0.0031)	0.0007	(0.0003)	0.0035	(0.0020)	-0.0194	(0.0142)
Income$_{t-1}$	0.0148	(0.1586)	0.8499	(0.0174)	0.4498	(0.1041)	1.4761	(0.7366)
Property Value$_{t-1}$	0.0118	(0.0147)	0.0018	(0.0016)	0.9359	(0.0096)	0.1401	(0.0682)
Minimum Wage$_{t-1}$	0.0010	(0.0008)	0.0002	(0.0001)	-0.0010	(0.0005)	0.9869	(0.0037)
Intercept	-0.0061	(0.0096)	0.0072	(0.0011)	-0.0076	(0.0063)	0.1136	(0.0448)

(e) Calculate the discounted profits for firm i from the perspective of the initial state for the alternative paths, $V^{b,s}(\tilde{\sigma}^b, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \tilde{\pi}_{it}^{b,s}$.

(f) Estimate the ex-ante value of being in state (N_1^b, X_1^b, Z_1^b) for firm i (when using the alternative policy against the estimimated policies of the rival firms) by averaging the discounted profits over all S paths: $\bar{V}^b(\tilde{\sigma}^b, \alpha) = \frac{1}{S} \sum_{s=1}^{S} \tilde{V}^{b,s}(\tilde{\sigma}^b, \alpha)$.

(g) Calculate the difference in the ex-ante valuations, $g_b(\hat{\sigma}, \alpha) = \bar{V}^b(\hat{\sigma}, \alpha) - \bar{V}^b(\tilde{\sigma}^b, \alpha)$.

2. Use the values $g_b(\hat{\sigma}, \alpha)$ for each of the B initial states and alternative policies to calculate the value of the minimum distance function, $Q(\alpha) = \frac{1}{B} \sum_{b=1}^{B} (\min\{g_b(\hat{\sigma}, \alpha), 0\})^2$.

A.3 Seemingly Unrelated Regressions (SUR) Model

To model the joint evolution of the exogenous state variables, we employ an SUR model. One justification for this approach is that all of these variables may be correlated at some level. For example, income and property value often move along similar trends. The SUR specification is:

$$
\begin{bmatrix} X_{1t} \\ X_{2t} \\ ... \\ X_{kt} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ ... \\ c_k \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & ... & A_{1k} \\ A_{21} & A_{22} & ... & A_{2k} \\ ... & & & \\ A_{k1} & A_{k2} & ... & A_{kk} \end{bmatrix} \cdot \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ ... \\ X_{kt-1} \end{bmatrix} + \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ ... \\ e_{kt-1} \end{bmatrix}
$$

where $E[e_t e_t'] = \Omega$ and where $c = (c_1, \ldots, c_k)$, $A = (a_{ij})$, and Ω are parameters to be estimated. Estimates of the intercepts c and the coefficients A are reported in Table 11 and estimates for the covariances are reported in Table 12.

A.4 Reduced Form Policy and Payoff Estimates

Table 13 contains the reduced form policy estimates from the first stage for each specification as described in Section 4.1. As indicated in the table we include either city fixed effects or city-specific drift terms, denoted η_m, and firm fixed effects or firm-specific drift terms in all specifications. For

Table 12: Estimated SUR Covariance Matrix

	Population$_t$	Income$_t$	Property Value$_t$	Minimum Wage$_t$
Population$_{t-1}$	2.2420	-0.0060	-0.0017	-0.3005
Income$_{t-1}$	-0.0060	0.0271	0.0011	-0.0102
Property Value$_{t-1}$	-0.0017	0.0011	0.9651	0.2625
Minimum Wage$_{t-1}$	-0.3005	-0.0102	0.2625	48.3756

Note: All entries have been multiplied by 10^3 for easier comparison.

specifications which include the Z process the inclusion of drift terms is denoted as "Z" while in the specification with only i.i.d. unobservables, the inclusion of fixed effects is denoted as "Yes". For both specifications including the Z process, we used 1000 particles to approximate the distribution of Z_{imt} for each firm i, market m, and time period t. Finally, Table 14 reports the remaining second-stage estimates, which are the reduced form portion of payoffs involving market characteristics.

A.5   Additional Simulation Analysis Details

To implement the model simulations and counterfactuals, we employ a similar forward simulation approach as in Benkard, Bodoh-Creed, and Lazarev (2010), which does not require one to solve a computationally intractable dynamic model. First, we estimate the different specifications (full model with Z process, model without Z process, model with Z process but no spillovers) and store the first and second stage estimates.

We then initialize the market characteristics (i.e., population, income, property value, and minimum wage) and market structure (i.e., the initial number of outlets for each chain) using data for the first year for each market. The unobserved and serially correlated Z process also needs to be initialized in our simulations. We draw the first period chain-market-specific $Z_{i,m,1}$'s from the corresponding steady-state distributions at time period $t = 1$ under the assumption of no size spillovers from $t = 0$ as by definition $N_{i,m,0} = 0$ for all firms. Using the estimated Z process, under these assumptions for each chain i in market m in the initial period $t = 0$, the stationary mean is $(\mu_i + \eta_m)/(1 - \delta_i)$, and the stationary variance is $\omega_i^2/(1 - \delta_i^2)$ (see e.g., Hamilton, 1994, p. 53).

Using the estimates, inferred policy functions from the first stage estimation, and SUR process for the exogenous market characteristics, along with the initializations, we then forward simulate the evolution of the number of stores and per-period profits across all markets m, for each of the specifications. In each market, we simulate 250 sample paths (with length of 36 years) given the initial market conditions, distribution of Z's, and inferred policy functions.

Table 13: Reduced Form Policy Estimates

Parameter	Z (Spillovers)		Z (No Spillovers)		No Z	
Population	0.0674	(0.0006)	-0.1457	(0.0030)	-0.1416	(0.0948)
Income	0.0759	(0.0010)	0.2019	(0.0017)	0.2081	(0.0197)
Property Value	-0.0328	(0.0004)	-0.1472	(0.0048)	-0.1398	(0.0486)
Grey Cup Host	0.3742	(0.0072)	0.1436	(0.0044)	0.1479	(0.0755)
Smoking Regulation	-0.1738	(0.0014)	-0.0810	(0.0015)	-0.1395	(0.1196)
Minimum Wage	-0.1746	(0.0017)	-0.1334	(0.0018)	-0.1252	(0.0412)
Population^2	0.0505	(0.0009)	0.0679	(0.0013)	0.0553	(0.0271)
Population $\times$ Income	0.0420	(0.0007)	0.0481	(0.0013)	0.0487	(0.0257)
Population $\times$ Prop. Value	-0.0611	(0.0005)	-0.0677	(0.0011)	-0.0646	(0.0325)
Population $\times$ Min. Wage	-0.0398	(0.0004)	-0.0539	(0.0011)	-0.0476	(0.0188)
Income^2	-0.0214	(0.0002)	-0.0219	(0.0003)	-0.0233	(0.0035)
Income $\times$ Prop. Value	-0.1038	(0.0012)	-0.1287	(0.0004)	-0.1323	(0.0088)
Income $\times$ Min. Wage	0.0359	(0.0006)	0.0334	(0.0006)	0.0354	(0.0062)
Property Value^2	0.1278	(0.0015)	0.1700	(0.0014)	0.1709	(0.0205)
Property Value $\times$ Min. Wage	0.0123	(0.0001)	0.0140	(0.0005)	0.0139	(0.0150)
Minimum Wage^2	-0.0150	(0.0002)	-0.0145	(0.0003)	-0.0146	(0.0088)
Own Z	1.0000	–	1.0000	–	–	–
Own $Z \times$ Population	0.2967	(0.0076)	0.0353	(0.0017)	–	–
Own $Z \times$ Income	-0.0055	(0.0001)	-0.0112	(0.0001)	–	–
Own $Z \times$ Prop. Value	-0.0006	(0.0000)	-0.0001	(0.0000)	–	–
Own $Z \times$ Min. Wage	0.0064	(0.0001)	0.0050	(0.0001)	–	–
Rival Z	-0.1856	(0.0015)	-0.1433	(0.0052)	–	–
Rival $Z \times$ Population	0.0045	(0.0000)	0.0083	(0.0002)	–	–
Rival $Z \times$ Income	0.0079	(0.0001)	-0.0114	(0.0003)	–	–
Rival $Z \times$ Prop. Value	-0.0023	(0.0000)	-0.0115	(0.0002)	–	–
Rival $Z \times$ Min. Wage	0.0379	(0.0004)	0.0079	(0.0002)	–	–
Cutoff 1 (ϑ_1)	-7.1358	(0.1725)	-4.0207	(0.1255)	-3.4663	(0.8414)
Cutoff 2 (ϑ_2)	-4.5462	(0.1044)	-3.1793	(0.1034)	-2.8467	(0.1905)
Cutoff 3 (ϑ_3)	-3.3214	(0.0969)	-2.3914	(0.1055)	-2.1811	(0.1437)
Cutoff 4 (ϑ_4)	0.9318	(0.0146)	0.8660	(0.0220)	0.8186	(0.0776)
Cutoff 5 (ϑ_5)	1.9484	(0.0212)	1.6257	(0.0246)	1.5274	(0.0734)
Cutoff 6 (ϑ_6)	2.5782	(0.0415)	2.0906	(0.0418)	1.9487	(0.0805)
Cutoff 7 (ϑ_7)	3.7805	(0.0928)	2.8829	(0.1073)	2.6372	(0.1204)
City Fixed Effects (η_m)	Z		Z		Yes	
Firm Fixed Effects (μ_i)	Z		Z		Yes	
Observations	5580		5580		5580	
Particles	1000		1000		-	
Log Likelihood	-3918.23		-4044.55		-4059.40	
AIC	8018.45		8233.10		8232.80	
BIC	8159.03		8344.33		8320.85	

Table 14: Reduced Form Payoff Estimates

Parameter	Z (Spillovers)		Z (No spillovers)		No Z	
Population ($\theta_{1,1}$)	0.0372	(0.0190)	0.0096	(0.0145)	-0.1169	(0.1197)
Income ($\theta_{1,2}$)	-0.0041	(0.0056)	-0.0040	(0.0083)	0.0319	(0.0348)
Property Value ($\theta_{1,3}$)	0.0139	(0.0110)	-0.0060	(0.0131)	-0.2162	(0.0616)
Grey Cup Host ($\theta_{1,4}$)	0.0034	(0.2181)	0.0910	(0.2968)	0.8161	(1.4234)
Smoking Regulation ($\theta_{1,5}$)	0.0476	(0.0344)	0.0488	(0.0392)	0.0036	(0.3305)
Minimum Wage ($\theta_{1,6}$)	0.0071	(0.0106)	0.0319	(0.0136)	0.1940	(0.0552)